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7 Modern Turns in Mathematics and Physics

PREFACE: NATURAL SCIENCE AND ETHICAL DISPOSITION

The boundary between wishing and choosing is defined by the limits of our power. As Aristotle says, we can wish "for impossible things, like [bodily] immortality," but we actively choose what we judge to be within our power.[1] Differing accounts of physical nature—specifically, differing accounts of the relation between the sensible and the intelligible—convey different implications concerning what is within our power, and thus different implications for our pattern of choices in life and related dispositions of will and appetite.[2] Here lies the moral significance of early modern philosophy, and its offspring, classical physics. In Bacon and Descartes, culminating in Newton, *laws of nature* replace natural forms and ends as the fundamental intelligibles of physical science, and

1. "[Choice] is not wish, although it seems near to it; for choice cannot relate to impossible things … but there can be wish for impossible things, like immortality …. But no one chooses such things, but only what he thinks can be brought about through his own efforts.… In general, choice seems to be concerned with the things that are up to us." *Nicomachean Ethics*, trans. Terence Irwin (Indianapolis, Ind.: Hackett, 1999), III.2.1111b20–30 (33–34), translation slightly modified [hereafter "*NE*"].

2. "The origin of action … is choice, and that of choice is desire and reasoning with a view to an end. Hence choice cannot exist either without intellect and thinking, or without ethical disposition; for good action and its opposite cannot exist without thinking and character" (*NE* VI.2.1139a31–35 [87]). "Each state of character has its own ideas of the noble and the pleasant" (*NE* III.4.1113a32 [37]). Irwin translation slightly modified in both quotations. "Do you suppose that the regimes arise 'from an oak or rocks' and not from the types of character of the men in the cities, which, tipping the scale as it were, draw the rest along with them?" Plato, *Republic* 544d–e, trans. Allan Bloom, *The Republic of Plato* (New York: Basic, 1991), 222.

imply a vast expansion of human power to predict, control, and transform natural processes and bodies—for the prolongation of life and "the relief of the human estate."[3] The latest phase of this project is synthetic biology and its apparently radical possibilities for the transformation of human life. There is even talk of achieving bodily immortality—something impossible on grounds of Aristotle's natural science of organic form, or soul, and its correlative, necessarily corruptible matter.[4]

Because of these connections, between accounts of physical nature and their implications—real or imagined—for the limits of our power, our patterns of choice, and resulting dispositions of will, desire and imagination, I doubt that it is possible to separate completely natural philosophy and metaphysics from ethics.[5] This is especially problematic today due to the difficulty of accurately discerning the range and limitations of modern science. Accordingly, the following presentation and comparison of three accounts of nature (Aristotelian, classical, and quantum) owes much to the thought of William A. Wallace on Aristotelian physics, Richard Kennington on early modern philosophy, and Jacob Klein on modern symbolic mathematics.[6]

3. Bacon, *New Organon*, ed. Fulton H. Anderson (Indianapolis, Ind.: Bobbs-Merrill, 1960), II.52 (267); see also I.88, I.129, II.1–6 (86, 118–19, 121–27). See *Wisdom of the Ancients* XIII, Proteus, in *The Works of Francis Bacon*, ed. J. Spedding, R. L. Ellis, and D. Heath (London: Longmans and Co., 1857–70; reprinted in Stuttgart-Bad Canstatt: Frohmann-Holzboog, 1963), 6:725–26. In *The Advancement of Learning*, Bacon refers to "the dignity and excellency of knowledge and learning in that whereunto man's nature doth most aspire, which is immortality or continuance" (*Works* 3:318). See also Descartes, *Discourse on the Method*, Part 6 (AT VI 61–62, CSM I 142–43), and *Principles* (French) III.44 (AT IXB 155, CSM I 255).

4. Stanley Shostak, *Becoming Immortal: Combining Cloning and Stem-Cell Therapy* (Albany: State University of New York Press, 2002).

5. "Every desire [*orexis*] … is for the object of desire [*orekton*], which is the starting point of the practical intellect. . . .But desire or imagination [may be] right and [may be] not right. . . .What always causes motion is the object of desire [*orekton*]; but this object may be either the good or the apparent good" (*De Anima* III.10.433a16–29; author's translation).

6. The philosophical comprehension of modern science in relation to premodern physics and mathematics is a central focus of their work. See especially William A. Wallace, *From a Realist Point of View* (Lanham, Md.: University Press of America, 1983); Richard Kennington, *On Modern Origins: Essays in Early Modern Philosophy*, ed. Pamela Kraus and Frank Hunt (Lanham, Md.: Lexington, 2004); Jacob Klein, *Greek Mathematical Thought and the Origin of Algebra*, trans. Eva Brann (Cambridge, Mass.: MIT Press, 1968).

INTRODUCTION: TECHNICAL ANALYSIS AND PHILOSOPHICAL INTERPRETATION OF PHYSICS

The principal device employed in this essay is a comparison between classical physics, on the one hand, and Aristotelian physics, on the other. From the contrast between them, two fundamental features of classical physics are brought out: species-neutrality, which concerns the relation between the intelligible and the sensible—a standard topic in philosophy—and physico-mathematical secularism, which concerns the relation between mathematics and natural science, is not a standard topic, and is explained in the following.[7] In keeping with the Preface on ethics, I note as well an immoderate disposition or attitude to nature—we could call it *transformism*—that arises in connection with early modern philosophy and classical physics, and that continues on into genetic science today, despite great differences between contemporary biology and classical physics.

Classical physics and its associated conception of the material world are conveniently exemplified by Newton's theory of the solar system and its generalization to the forces-and-particles model of the universe, a vast mental construct at the heart of which is the concept of *trajectory*—the path in space of a body moved according to laws of motion and force, starting from a given position with a given velocity.[8] Certain basic and very general characteristics of this Newtonian account are found throughout all of prequantum physics, including thermodynamics, the field physics of Maxwell, the relativity physics of Einstein, and the recently developed non-linear dynamics or "chaos theory" originally discovered by Poincaré.[9] These characteristics are well described by Louis de Broglie,

7. I take the term "species-neutrality" from Kennington, *Modern Origins*, 25, and private conversations. I take the term "secularism" in relation to mathematical physics from François De Gandt: "Newton claimed to treat forces in a purely mathematical mode; by deferral, which in a sense turned out to be final, he left in suspense the properly philosophical or physical questions concerning the causes of gravitation and the ontological reality of force. This neutrality (or 'secularism') of centripetal force in face of the controversies on the cause of gravitation is the essential characteristic of the new science." *Force and Geometry in Newton's* Principia, trans. Curtis Wilson (Princeton, N.J.: Princeton University Press, 1995), x–xi. Physico-mathematical secularism concerns the question whether mathematical objects and physical objects differ and, specifically, whether it matters.

8. The starting position and velocity of the body are "given" in two senses: first, as stipulated or set down by the theoretical physicists in order to initiate the calculation of the trajectory under the equations of motion (to see how it behaves); second, as measured by the experimental physicists using instruments (like meter sticks and clocks).

9. Henri Poincaré, *Science and Method*, trans. Francis Maitland (New York: Dover, 1952),

as discussed in part 1, below, in which the basic theme of mathematics and nature, ancient and modern, is introduced. Part 2 presents the Newtonian trajectory calculation for gravitational force as a concrete example of de Broglie's general description of classical physics. Part 3 then elaborates the species-neutrality of Newton's law of gravitational force. Part 4 treats Newton's proposed generalization of the gravitational theory of the solar system to the forces-and-particles model of the universe, a mental construct with radical implications for the meaning of nature, one of which is transformism, discussed in part 5. Part 6 then turns to an alternative understanding of nature, Aristotelian physics, and presents the sharp contrast between it and classical physics. Throughout, by "classical physics" I mean all post-Aristotelian, mathematical, and experimental physics, with the exception of quantum theory. I treat the latter mainly indirectly, in part 7, by means of brief comparisons to Aristotelian and classical physics. Against the background of classical physics, quantum physics appears indeed to be a radical departure. But, as I argue in the following, classical physics is itself a radical departure from the preceding Aristotelian understanding of nature.[10] This is hardly to say that Aristotelian natural philosophy can provide the adequate philosophical compre-

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67–69. Chaotic dynamics falls within classical physics because the mathematical models (non-linear differential or difference equations) hypothesized to account for certain unpredictable physical phenomena (e.g., a forced pendulum, turbulent flow, weather patterns) yield deterministic trajectories, as discussed in part 1, below. What distinguishes chaotic dynamics within classical physics is the following secondary but significant point concerning the initial data, i.e., the numerical values inputted to the model (the first sense of "given" in note 8) that specify a particular deterministic trajectory: due to sensitive dependence on initial conditions—a feature of non-linear equations—no matter how close the numerical values with which two modeled trajectories begin, they (the two trajectories) diverge so rapidly that even the most precise *empirical* measurement of initial data, e.g., on turbulent flow, a weather system, a forced pendulum (the second sense of "given" in note 8) would still contain within its tiny but necessarily finite error interval differences in numerical value that would determine trajectories so rapidly diverging that the physical process would *appear* random, i.e., under the same empirically measured conditions, very different physical changes occur no matter how precise the measurements. Thus, any systems in nature governed by non-linear interactions of this type (subject to sensitive dependence on initial conditions) would be fully deterministic but quite unpredictable in their behavior. Hence the name "deterministic chaos" is often used to describe the field of non-linear dynamics; see Heinz Georg Schuster, *Deterministic Chaos* (Weinheim: VCH Verlagsgesellschaft, 1989), 1–5. For a good nontechnical presentation, see David Ruelle, *Chance and Chaos* (Princeton, N.J.: Princeton University Press, 1991), also James Gleick, *Chaos: Making a New Science* (New York: Viking Penguin, 1987). See note 85, below, on the philosophical significance of chaos theory.

10. "The design of reality in classical physics contains greater enigmas than your so-called quantum mystery." Kurt Riezler, *Physics and Reality: Lectures of Aristotle on Modern Physics at an International Congress of Science* (New Haven, Conn.: Yale University Press, 1940), 25.

hension of quantum physics. The quantum phenomenon of non-locality or entanglement is a challenge for any philosophy of nature today, and, historically, the Aristotelian doctrine of the essential heterogeneity of terrestrial and celestial matter and motion was an error that impeded the progress of science.[11] Nevertheless, I believe that the Aristotelian back-

11. Non-locality or entanglement refers to a remarkable effect—instantaneous action at a distance to arbitrary range—predicted by quantum physics and highlighted in the famous 1935 exchange between Bohr and Einstein, Podolsky, and Rosen (EPR), over the completeness of the Copenhagen Interpretation. Einstein, Podolsky, and Rosen, "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?" *Physical Review* 47 (May 15, 1935): 777–80; Bohr, "Can Quantum-Mechanical Description of Physical Reality be Considered Complete?" *Physical Review* 48 (October 15, 1935): 696–702. We can get a sense of the issue against the background of Einstein's firmly classical conception of what physics is and ought to be: "I cannot seriously believe in [quantum mechanics] because the theory cannot be reconciled with the idea that physics should represent a reality in time and space, free from spooky actions at a distance." Einstein to Born, 1947, in Max Born, *The Born-Einstein Letters* (New York: Walker and Co., 1971), 158. "But on one supposition we should, in my opinion, hold absolutely fast: the real factual situation of system S_2 is independent of what is done with the system S_1, which is spatially separated from the former …. (One can escape from this conclusion only by either assuming that the measurement of S_1 (telepathically) changes the real situation of S_2 or by denying independent real situations as such to things which are spatially separated from each other. Both alternatives appear to me entirely unacceptable." Einstein, "Autobiographical Notes" (1949), in Paul Arthur Schilpp, *Albert Einstein Philosopher-Scientist* (La Salle, Ill.: Open Court, 1969), 1:85. What Einstein means is that physical influences from one position in space (that of S_1) can produce an effect at another position (that of S_2) only by propagating temporally through the intervening space. Einstein's special relativity limits the propagation speed to that of light. For Einstein (as for many), instantaneous action at a distance is by nature impossible. Schroedinger clearly foresaw the prediction of "spooky actions at a distance" entailed by his equation for the wave function. Writing shortly after the EPR-Bohr exchange, he introduced the term "entanglement": "When two systems, of which we know the states by the respective representatives, enter into temporary physical interaction due to known forces between them, and when after a time of mutual influence the systems separate again, then they can no longer be described in the same way as before, viz. by endowing each of them with a representative of its own [independent of the other]. I would not call that *one* but rather *the* characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought. By the interaction, the two representatives (or ψ-functions) have become entangled. To disentangle them we must gather further information by experiment …. After reestablishing one representative by observation, the other one can be inferred simultaneously. In what follows the whole of this procedure will be called *the disentanglement*. Its sinister importance is due to its being involved in every measuring process …. Attention has recently been called [Einstein, Podolsky, and Rosen, *Phys. Rev*. 47 (1935): 777] to the obvious but very disconcerting fact that even though we restrict the disentangling measurements to *one* system, the representative obtained for the *other* system is by no means independent of the particular choice of observations which we select for that purpose and which by the way are *entirely* arbitrary. It is rather discomforting that the theory should allow a system to be steered or piloted into one or the other type of state at the experimenter's mercy in spite of his having no access to it. This paper does not aim at a solution of the paradox, it rather adds to it, if possible." Schroedinger, "Discussion of Probability Relations between Separated Systems," *Proceedings of the Cambridge Philosophical Society* 31, no. 1 (January 1936): 555–56. EPR correlations were confirmed by the Aspect

ground is valuable, perhaps indispensable, for the philosophical interpretation of physics, both classical and quantum. An example is presented in part 8, on physico-mathematical secularism.

Part 1—Basic Characteristics of Classical Physics: De Broglie's Synoptic Description

Louis de Broglie provides a remarkable overview of classical physics in his 1955 *Physics and Microphysics.* The lengthy passage quoted below describes basic characteristics common to the three great theories of classical physics—mechanics, electromagnetism, and thermodynamics—as well as Einsteinian relativity and chaotic dynamics.

With [Cartesian] coordinates of space and time [x, y, z, t], classical mathematical physics was in a condition to represent in a precise way the succession of phenomena which our senses allow us to verify around us.

From that moment a way opened quite naturally before theoretical physics and it boldly entered upon it. It was thought that all evolution of the physical world must be represented by quantities [e.g., the position and velocity of a discrete particle, or the intensity of a continuous field] localized in space and varying in the course of time. These quantities must render it possible to describe completely the state of the physical world at every instant, and the description of the whole of nature could thus be given by figures and by motions in accordance with Descartes's programme. This description would be entirely carried out with the aid of differential equations … enabling us to follow the localization and the evolution in the course of time of all the quantities defining the state of the physical world. A magnificent conception for its simplicity and confirmed by the successes which it has achieved for a long time! It sustained and orientated all the efforts of the great schools of mathematical physics of the nineteenth century.

Assuredly not all scientists agreed to this description of the world by figures and movements exactly in the same way. Some with lively and concrete imagination sought to picture the elements of the material world so as to make the phenomena observed by our senses flow from the existence and movements of

experiments in 1982. As of 2008, they have been detected to a range of 18 km with a speed that could be no less than ten thousand times that of light. D. Salart, A. Baas, C. Branciard, N. Gisin, H. Zbinden, "Testing Spooky Action at a Distance," *Nature* 454 (August 14, 2008): 861–64. On the now-problematic relation between this amazing effect and special relativity, see Tim Maudlin, *Quantum Non-Locality and Relativity* (Malden, Mass.: Blackwell, 2002).

atoms or of corpuscles too small to be directly observed; they wanted to dismantle the machine to see all the wheels functioning. Others, more cautious and above all endowed with a more abstract mind, wanted to content themselves by uniquely representing phenomena by means of directly measurable quantities, and mistrusted the hypotheses—in their eyes too speculative and useless—of the atomists. And whereas the atomists were thus boldly advancing, opening new ways and allowing science to make astonishing progress, the energeti[ci]sts, impeded by their more formal and timid methods, retained a certain advantage from the conceptional point of view when they denounced what was simple and a little naïve in the pictures invoked by their bold rivals. But, without being aware of it, both [the atomists and the energeticists] admitted a … number of common postulates of which the future was to prove the frailty.

They were, in fact, agreed in admitting the validity of the abstract framework of space and time, the possibility of following the evolution of the physical world with the aid of quantities well located in space and varying continuously in the course of time, and the legitimacy of describing all phenomena by groups of differential equations. If the energeti[ci]sts, like Pierre Duhem, refused to allow the intervention everywhere of the "local movement" which could be represented by a displacement of parts, they fully admitted the consideration of "general movements" defined more abstractly by the variations of quantities in the course of time. In spite of their differences of view on the manner of carrying out this program, all theorists were then in agreement in representing the physical universe by well-defined quantities in the framework of space and time and subject to differential equations.

The differential equations … of classical mathematical physics have the common character of allowing us to follow rigorously the whole evolution of the phenomena which they describe, if we suppose that there are certain known data relative to an initial state corresponding to a particular value of time. From this there was deduced the possibility of establishing a kind of inevitable interconnexion of all the phenomena, and thus was reached the conception of a universal determinism of physical phenomena. It is not my purpose to examine from the philosophical point of view the idea of universal determinism, and I have not to ask myself, for example, if the mind, which, after all is said and done, is the creator of mathematical physics, could recover its place in a nature conceived of in such a rigid manner. What is certain is that physical phenomena, in so far as they were exactly represented by the differential equations of classical physics, were submitted to a very precisely defined determinism.

Classical physics thus represented the whole physical universe as projected with absolute precision into the framework of space and time, evolving from it according to the laws of an inexorable necessity. It completely set aside the means used to arrive at a knowledge of the different parts of this vast mechanism for, if it recognized the existence of experimental errors, it only saw in them a result of the lack of precision of our senses and of the imperfection of our techniques, and accepted the possibility of reducing them indefinitely, at least in principle, by an adequate improvement in our methods. All these representations rested essentially on the classical ideas of space and time; for a long time they appeared sufficient for a description of the evolution of the material world.[12]

From this long passage, I distill three basic characteristics of all classical physics. Implicit therein is a notion of the relation between human mind in its knowing activity and the physical world that is to be known. The three characteristics and the associated mind-world relation stand opposed, on the one hand and in one way, to Aristotelian physics, and, on the other and in a different way, to quantum physics. The three are *continuity of space, time, and motion*; *spatio-temporal imageability of fundamental processes*; and *universal determinism*. Let us consider each in turn.

First, *continuity of space, time, and motion* is a shorthand formula for what de Broglie more accurately describes: continuity in the spatio-temporal variation of physical quantities such as the position, r(t), and velocity, v(t), of a discrete particle, or the intensity of a continuous (say, electrical) field, E(r,t).[13] As an apt example of what is physically meant by continuity, consider a body in motion that suddenly disappears from its present position and instantaneously reappears at a different position; this would be absurd, as in a dream. A planet, for example, does not change its distance from the sun by suddenly disappearing from its present position and instantaneously reappearing on an orbit with a different radius. The motions of planets and,

12. Louis de Broglie, *Physics and Microphysics*, trans. Martin Davidson (New York: Harper Torchbooks, 1960), 116–18. "Descartes's programme" refers, as de Broglie puts it, to "the possibility of describing natural phenomena by figures and by motion in the framework of space and time … capable of allowing, always and everywhere the establishment of rigid and precise ties of inevitable succession amongst all natural phenomena" (110). This project is clearly set out by Descartes in *World* and *Principles of Philosophy*; *Principles* 2.64 provides an especially compact formulation: "The only principles that I accept or require in physics are those of geometry and pure mathematics; these principles explain all natural phenomena, and enable us to provide quite certain demonstrations regarding them" (AT VIIIA 78, CSM I 247), taking "geometry" here to mean his own analytical geometry applied to numerical magnitudes possessing physical dimensions, as described in Rule 14 (AT X 447–50, CSM I 62–64), and familiar today as length, mass, time, charge, etc.

13. On discrete particles; or, for most applications, positions and velocities of many particles, $r_1, \dots r_N, v_1, \dots v_N$. As N increases, probability theory is then brought to bear, e.g., in kinetic theory of gases and statistical mechanics.

more generally, the local motions of bodies, are continuous; there are no jumps. But jumps do occur among the electronic states or orbitals of atoms and molecules; it is part of their distinctive form of stability, as discussed in part 7, below. The classical continuity of space, time, and motion thus contrasts sharply with the state transitions of quantum physics.

What about Aristotle; does he deny continuity of local motion in favor of some sort of quantum jumps? He does not. For, in *Physics* IV.11, Aristotle says: "Since a moving thing is moved from something to something and every magnitude is continuous, the motion follows the magnitude; for through the magnitude's being continuous, the motion too is continuous, and through the motion the time." And again in *Physics* VI.1: "It is impossible for anything continuous to be made of indivisible things; for example, a line cannot be made of points, if the line is continuous and the point indivisible [*Metaphysics* 1016b27]. And it belongs to the same argument for a magnitude, a time, and a motion to be composed of and divided into indivisibles, or none of them."[14] Finally, in *Physics* VI.2: "every magnitude is divisible into magnitudes (for it has been shown that it is impossible for anything continuous to be made of uncut-able parts, and every magnitude is continuous)."[15] Therefore, the opposition between classical physics and that of Aristotle concerning space, time, and motion must be on a different level, namely, that of the relation between human mind in its knowing activity and the physical world that is to be known.

What must be appreciated in de Broglie's exemplary expression of the self-understanding of classical physics is that mind is there looking not directly at the world but rather at the products of its own constructive activity: elaborate structures of real-numerical variable magnitudes of which the variables, x, y, z, and constants, a, b, c, etc., of Descartes's *Geometry*, essentially our analytic geometry, are the prototype. The material world external to this sophisticated mental artifice is admitted as *known* only guardedly, under the carefully controlled conditions of the experiment: does the numerical result of the empirical measurement *match* the mathematically predicted numerical value to within current limits of experimental precision (the error intervals)? If so, then the conceptual structure of variable quantities possessing physical dimensions, the equations and graphs derived from the laws of physics, that is, the

14. *Physics* IV.11.219a11–13, VI.1.231a24–25, b18–20; trans. Joe Sachs, *Aristotle's* Physics: *A Guided Study* (New Brunswick, N.J.: Rutgers University Press, 1995), 121, 147–48.

15. *Physics* VI.2.232a23–25 (149).

model that has been proposed by us to nature is accepted as corroborated.[16] If not, then more adequate models based on revised equations or new laws are to be constructed; we are on our way into the standard philosophy of hypothetico-deductive science that is available in many textbooks. For our purposes here, let us note that the mind-world relation characteristic of physics has two components, the mathematical model and the extra-mathematical reality to be modeled: they "transact" in the "common currency" of number, calculated and measured.

The world-conception of classical physics described by de Broglie assumes the progressive improvement of the numerical precision of experimental measurements, and thus of the match between filtering mind and the filtered ("true") world. It thus assumes the complete adequacy of real number to nature, as discussed in part 8, below. The point for now is simply that this is not the type of mind-world relation that is found in Aristotle's *Physics*. There, intellect looks directly through the senses at the kinds of beings that make up the world: the elements (the land, the waters, the weather), plants, animals, artifacts and the human beings that produce them, and, above it all, the celestial bodies, which appear to move so differently from all the rest. Each of these beings is a whole whose parts are related in various ways, and all are subject to being moved, to acting and being acted upon, in various ways according to the causes judged to be necessary in order to fit speech to the phenomena, beginning with ourselves who bespeak the phenomena.[17] Aristotle's way of thinking together the objects of his science of nature is as follows: "what moves is a mover of something moved, and what is moved is moved by something moving it, and there is no motion apart from things

16. "Mathematical physics … is bent on *matching* the consequences derived mathematically from hypotheses with observations dictated by these hypotheses.… It is not its business to say what, for example, gravitation, or electromagnetism, or energy *is*, except by establishing in a symbolic-mathematical formula the relations that bind these entities (if it is at all permissible to use this word) to observable and mathematically describable magnitudes." Jacob Klein, "On Precision," in *Jacob Klein: Lectures and Essays*, ed. Robert B. Williamson and Elliott Zuckerman (Annapolis, Md.: St. John's College Press, 1985), 305–6, and, above all, Kant: "reason has insight only into that which it produces after a plan of its own, and … it must not allow itself to be kept, as it were, in nature's leading-strings but must itself show the way with principles of judgment based upon fixed laws, constraining nature to give answer to questions of reason's own determining." *Critique of Pure Reason*, trans. Norman Kemp Smith (New York: St. Martin's Press, 1965), B xiii (20).

17. See especially *Parts of Animals* I.1.639a1–41a18 for the self-referential character of Aristotle's natural philosophy.

[*ouk esti de tis kinêsis para ta pragmata*]."[18] Motion, that which is mobile, and causes of motion must be thought together, along with the time and (in local motion, increase and decrease) the magnitude traversed. Finally, for Aristotle, the meaning of physical motion (*kinêsis*) cannot be adequately expressed without using the words most characteristic of his understanding of nature: act and potency, *energeia* and *dunamis*:

motion is the actuality of the potentially being as such [*hê tou dynamei ontos entelecheia hê toiouton kinêsis estin*].... it cannot be placed in an unqualified way either under the potentiality or under the actuality of things [*oute eis dunamin tôn ontôn oute eis energeian*].... a motion [then] is sort of an actuality [*energeian men tina einai*] ... such as we have stated, difficult to grasp but capable of existing.[19]

This is far from the representation of motion as position, x(t), velocity, v(t) = dx/dt, etc. There, the symbols (x, v, t, dx, dt) possess a kind of objectivity, or being in their own right, deriving sense or meaning through their membership in the system of signs constituted by the axioms and binary operations of algebra prior to, and independently of, any possible application to the material and mobile beings that we see with our eyes and point at with our fingers.[20] For us, there *is* motion—mathematical "motion"—apart from things.[21]

18. *Physics* III.1.200b32–34 (73). The passage goes on to say that there is no change or motion outside the categories of substance, quantity, quality, or place. See also *Physics* II.2.193b23–194a15 and *Metaphysics* VI.1.1025b29–26a4: mathematical objects are separable from motion (*kinêsis*) and matter, but motion is not separable from matter.

19. *Physics* III.1.201a11–12, III.2.201b29–30, 202a2; trans. Hippocrates G. Apostle, *Aristotle's* Physics (Grinnell, Iowa: Peripatetic Press, 1980), 43–45.

20. "Nothing but the internal connection of all the concepts, their mutual relatedness, their subordination to the total edifice of science, determines for each of them a *univocal* sense [cf. *Metaphysics* IX.6.1048a37-b10, on the analogical sense of *energeia*] and makes accessible to the understanding their only relevant, specifically scientific content.... Two things ... constitute the heart of the symbolic procedure: ... identify[ing] the object represented [e.g., "two"] with the means of its representation ["2"], and ... replac[ing] the real determinateness of an object with a *possibility* of making it determinate, such as would be expressed by a sign [e.g., x] which, instead of *illustrating* a determinate object [e.g., thirty units], would *signify* possible determinacy [e.g., in an equation, 2x + a = b]." Klein, *Greek Mathematical Thought*, 121, 123. "Because the mere perceptual content of the signs involved in symbolic mathematics [e.g., 2, x] is insufficient to establish their mathematical significance, this significance must somehow be stipulated by the calculative method employed [the rule-governed method of designating and manipulating sense perceptible signs to perform 'calculations' with general mathematical objects], only after which the sense-perceptible signs can become known as 'symbols.'" Burt Hopkins, *The Origin of the Logic of Symbolic Mathematics: Edmund Husserl and Jacob Klein*, §140. An excellent summary of algebra for physicists is provided by Richard P. Feynman, Robert B. Leighton, and Matthew Sands, *The Feynman Lectures on Physics* (Reading, Mass.: Addison-Wesley, 1963), vol. 1, chap. 22.

21. "We commence with a chapter on *Motion*, a subject totally independent of the

Being detached or separate from material being, the mental apparatus of our formal-symbolic mathematics is like a universal and demiurgic tool that, as such, is separate from (not bound into, not a part of) the material to be worked on and, therefore, is available in advance for application to any material. In his review of Jacob Klein's *Lectures and Essays*, with its distinctive emphasis on the Greek understanding of number, *arithmos*, Richard Kennington provided the following apt formulation: "The Cartesian *numerus*, in contrast to the *arithmos*, cannot function as the bond of our world-relatedness; or rather its lack of such relatedness permits what is other than man to be understood as 'world' and Cartesian man to be world-less."[22] Thus, whatever we are, we are free and powerful. I note that, according to Klein, it is Plato for whom *arithmos* is the bond of our world-relatedness; for Aristotle, it is soul.[23] But Descartes transforms soul, too.

A sign of the difference between Aristotle's physics and classical physics is de Broglie's reference to "coordinates [x, y, z, t . . .] the abstract framework [the classical ideas] of space and time." As Klein states it at the conclusion of his account of the concept of number in Descartes:

> In Descartes' thinking, the dignity of representing the substantial "being" of the corporeal world accrues to extension precisely by reason of its *symbolic* objectivity within the framework of the *mathesis universalis*. Only at this point [Descartes's invention of general magnitude as the fundamental term of physics] has the conceptual basis of "classical" physics, which has since been called "Euclidean space," been created. This is the foundation on which Newton will raise the structure of his mathematical science of nature.[24]

existence of *Matter* and *Force*." William Thomson and Peter Guthrie Tait, *Treatise on Natural Philosophy* (Oxford: Clarendon Press, 1857), v. Thomson and Tait was a standard textbook of physics in the nineteenth century. A contemporary textbook is that of David Halliday and Robert Resnick, *Physics* (New York: John Wiley and Sons, 1978): "Mechanics, the oldest of the physical sciences, is the study of the motion of objects. . . . When we describe motion we are dealing with that part of mechanics called *kinematics*. When we relate motion to the forces associated with it and to the properties of the moving objects, we are dealing with *dynamics*" (30).

22. Richard Kennington, book review of Jacob Klein, *Lectures and Essays*, *Review of Metaphysics* 41, no. 1 (September 1987): 144–49, here 147. According to Klein, it is Plato for whom *arithmos* is the bond of our world-relatedness (*Lectures and Essays*, chap. 3). For Aristotle, rational soul is the bond, e.g., *De Anima* III.5.429a27, III.8.431b21.

23. E.g., *De Anima* III.5.429a27, III.8.431b21.

24. Klein, *Greek Mathematical Thought*, 211. The quotation marks indicate that "Euclidean space" is inaccurately so called in the sense that Euclidean geometry is not coordinate

This concludes my comment on the first of the three basic characteristics of classical physics: continuity of space, time, and motion.

The second basic characteristic is *spatio-temporal imageability of fundamental processes.* It means that we can always project in our mathematical imagination and express on paper Cartesian spatial coordinate axes (x, y, z) and then picture the relevant physical quantities—magnitudes with dimensions of length, mass, charge, time, and their combinations—varying in that space with time, t. In particular, we can know, from the data and the model, what is going on *inside* any physical system—an atom, an organism, a laboratory (e.g., two-slit interference) apparatus—even though the mind is looking mainly at the products of its own activity![25] We can imagine, for example, a particle with a precise position given by its Cartesian coordinates, x(t), y(t), z(t), and precise velocity with components, $v_x(t)$, $v_y(t)$, $v_z(t)$, for any time, t, moving on what is thereby defined as its *trajectory.* Or we can imagine (slightly more abstractly) the intensity of an electric field, **E**(x, y, z, t), varying in both spatial position, x, y, z, and time, t. This way of using the mind is assumed to be fully adequate to the nature of things; the common currency of number or, more precisely, numerical magnitude is taken as sufficient for all transactions between the mathematics (equations and calculation) and the physics (measurement and matching). This assumption does not entail the rather crude Cartesian *identification* of physical objects with mathematical objects, but rather the more sophisticated assumption that, whatever may be the differences between mathematicals and physicals, such differences can make no difference for the conduct of our (now thoroughly mathematical) physics.[26] In

(numerical) geometry—that is the achievement of Descartes (and Fermat). See Klein, "The World of Physics and the 'Natural' World," in *Lectures and Essays*, 21. In post-Newtonian physics, however, "Euclidean space" is often used in contradistinction to coordinate spaces with other metrics or distance functions, e.g., the Minkowski space of special relativity and the Riemannian space of general relativity. Descartes's coordinatization of geometry thus opens doors to remarkable physical theories whose development would be difficult to imagine without the enabling Cartesian foundation.

25. De Broglie's description of the atomists is apt: "they wanted to dismantle the machine to see all the wheels functioning." With the turn of the nineteenth century, the more phenomenological and cautious energeticists (Mach, Duhem, Poincaré, Ostwald) were vanquished by the rapidly mounting body of molecular theory corroborated by experimental evidence, e.g., Einstein on Brownian motion. See Jean Perrin, *Les Atomes* (Paris: Librarie Felix Alcan, 1913).

26. "I conceive its [matter's] extension ... not as an accident, but as its true form and essence." Descartes, *The World*, chap. 6, (AT XI 36, CSM I 92). "I recognize no matter in corpo-

other words, the natural-philosophic question of the difference between mathematical objects and physical objects can be suspended; let it be a private matter. This is what I mean by "physico-mathematical secularism," the subject of part 8, below.

The third basic characteristic, *universal determinism*, means that, through the equations of motion, the numerical values of the relevant quantities at one instant of time, t, or position, x, in space enable us to calculate the values of those quantities at the next instant of time, $t + \Delta t$, or adjacent position in space, $x + \Delta x$, and the next, on into future time and distant space. No other type of causality, beyond initial data or boundary conditions and equations of motion, is needed to account for all natural phenomena. But, as should now be clear, this determinism—first made explicit by Laplace—is based in the mathematics, that is, it is a feature of the mathematical description.[27] Its alleged universality in physics is a philosophical claim inspired by particular instances of success, to wit, *to the extent that* the behavior of a physical system, for example, the solar system, is found by observation or experiment to match the mathematical predictions of the model to within given limits of precision, we say that the physical system thereby modeled is deterministic. The successes of classical physics on various classes of deterministic phenomena (celestial mechanics, electromagnetism, thermodynamics, vibrations and waves in material media) were stunning, but they were always empirical and particular. And so the claim for *universal* determinism (of *all* natural phenomena) was a matter of the disposition and imagination of scientists and philosophers, not scientific warrant.

To state essential conclusions thus far: within the self-understanding of classical physics, strong claims (for continuity, imageability, determinism) about the transparency and, as we shall see, the malleability of nature are made based on (1) the mind's view of its own conceptual constructions and (2) a disposition to ascend from particular scientific results to universal philosophic claims. Let us illustrate the classical conception of the physical world by reviewing the basic logic of the Newtonian calculation of the trajectory of a body moved under gravitational force. This will exemplify de Broglie's account, enable us to understand species-neutrality,

real things apart from that which the geometers call quantity." *Principles* 2.64 (AT VIIIA 78–79, CSM I 247); also *Principles* 4.187 (AT VIIIA 314–15, CSM I 279).

27. Laplace, *A Philosophical Essay on Probabilities*, trans. F. W. Truscott and F. L. Emory (New York: Dover, 1951), 4, 6.

and prepare the generalization from the Newtonian theory of the solar system to the forces-and-particles model of the entire universe.

PART 2—EXAMPLE: ESSENTIAL LOGIC OF THE NEWTONIAN TRAJECTORY CALCULATION

Our goal in this section is not to become proficient classical celestial "mechanics" (able to solve actual problems of motion in the solar system) but rather to be able to *see the reason* for Newton's key achievement: the predictive calculation of the trajectory of a body in motion under gravitational force, for example, a planet or a comet around the sun. To this end, I follow not the current textbook approach (oriented to the training of successful researchers) but the more insightful description of Einstein and Infeld.[28]

The problem to be solved is this: given by empirical observation the position, $\mathbf{r}_0$, and velocity, $\mathbf{v}_0$, of a planet or comet relative to the sun at a given initial time, t_0, to derive the position and velocity, $\mathbf{r}_1$, $\mathbf{v}_1$, at a later time, t_1, without further recourse to observation.[29] The principles governing the calculation are two: Newton's second law of motion, and his law of universal gravitational force. We must examine the role played by each and recognize the crucial significance of the gravitational force law.

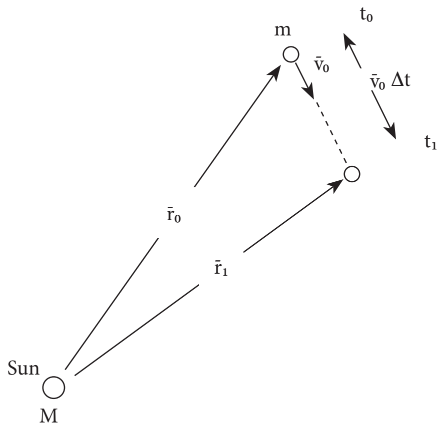
The conceptual framework of space is, in this case, the two- dimensional plane with the sun, of mass M, represented at the origin of coordinates.[30] We represent the initial position and velocity vectors, $\mathbf{r}_0$, $\mathbf{v}_0$, of the planet or comet, of mass m, as shown in figure 1. Can anything at all be predicted from our two pieces of initial data? The answer is, yes, approximately. Knowing $\mathbf{v}_0$, the initial velocity, we can *estimate* the

28. Albert Einstein and Leopold Infeld, *The Evolution of Physics* (New York: Simon and Schuster, 1968), 9–30. This derivation will be essential for part 7, on the breakdown of the trajectory concept.

29. We use the standard vector notation for position and velocity in Cartesian coordinates, x, y, z, and time, t: $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$, and $\mathbf{v}(t) = (dx/dt)\mathbf{i} + (dy/dt)\mathbf{j} + (dz/dt)\mathbf{k}$, where $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ are unit vectors along the x, y, z axes. The initial position and velocity, $\mathbf{r}(t_0) \equiv \mathbf{r}_0$, $\mathbf{v}(t_0) \equiv \mathbf{v}_0$, are the "known data relative to an initial state corresponding to a particular [initial] value of time [t_0]" referred to by de Broglie.

30. Angular momentum, $\mathbf{L} = \mathbf{r} \times m\mathbf{v}$, is conserved in the motion of a body of mass, m, under any central force, thus for an inverse-square gravitational force. Therefore, the position, $\mathbf{r}$, and the velocity, $\mathbf{v}$, of the moved body must lie in a fixed plane. (I am assuming the masses are known, a big assumption, in order to focus on the most essential point.)

FIG. 1



position, $\mathbf{r}_1$, of the planet or comet a short time later, at time t_1, by taking its motion as rectilinear (even though it is necessarily curvilinear due to the gravitational force of the sun). This is erroneous, but we can make the error as small as we like by making the time interval, $\Delta t = t_1 - t_0$, smaller and smaller. So we "move" the planet or comet, that is, we move the point that represents it in our figure, in the direction of $\mathbf{v}_0$ a distance that represents $\mathbf{v}_0\Delta t$ in the solar system. For example, if we are studying the motion of Mars, whose mean orbital speed is 24.1 km/sec, then, picking $\Delta t = 1$ sec, we would plot its new position at a distance in the direction of vo representing 24.1 km in space from its initial position. We have now derived from the initial data, $\mathbf{r}_0$, $\mathbf{v}_0$, the approximate position, $\mathbf{r}_1$, of the body at time t_1.

But what is the new velocity, $\mathbf{v}_1$? As noted, a planet or comet under the gravitational attraction of the sun moves on a curved path (it must, by Newton's first law). Thus the direction of the velocity vector that we use to represent its motion is constantly changing. To estimate the new velocity, $\mathbf{v}_1$, at time t_1, we must find the change in velocity of the body, $\Delta\mathbf{v}_{01}$, that occurs during the first time interval, Δt, from t_0 to t_1. Then we can construct, by vector addition, $\mathbf{v}_1 = \mathbf{v}_0 + \Delta\mathbf{v}_{01}$. This would complete our prediction of the new position *and* velocity of the planet or comet a short time, Δt, into the future. This may seem like a small step, because

Δt is small, for example, one second, but if we can repeat this procedure for the next time interval, from t_1 to t_2, deriving $\mathbf{r}_2$ and $\mathbf{v}_2$ from $\mathbf{r}_1$ and $\mathbf{v}_1$, and so on, then we will have made a world-historic revolution in human thought. The focus of our attention is, therefore, on $\Delta\mathbf{v}_{01}$: how is it determined? This is where Newton's law of universal gravitation comes into play; it is the crucial link in the logic of the trajectory calculation—linking $\Delta\mathbf{v}_{01}$ to $\mathbf{r}_0$—without which our procedure would stop dead after the estimate of r1.

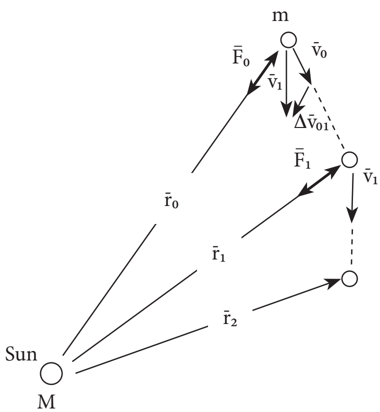
The second law of motion relates the net force on a body of mass m to its resulting acceleration or time rate of change of velocity: $\mathbf{F} = m\Delta\mathbf{v}/\Delta t$, in the limit as $\Delta t \to 0$. But what *is* the net force on the planet or comet under study? It is the law of gravitation that relates the known distance, $\mathbf{r}_0$, to the net force exerted by the sun on the planet or comet at time t_0.[31] Thus combining the law of gravitation with the second law of motion enables the determination of the (approximate) change in velocity, $\Delta\mathbf{v}_{01}$. The direction of $\Delta\mathbf{v}_{01}$ is that of the force, toward the sun, and its magnitude is calculated as follows (figure 2): $-GMm/r_0^2 = m\Delta v_{01}/\Delta t$. Therefore, $\Delta v_{01} = -GM\Delta t/r_0^2$, in which all the quantities on the right hand side are known. Having thus determined $\Delta\mathbf{v}_{01}$, we have by vector addition, $\mathbf{v}_1 = \mathbf{v}_0 + \Delta\mathbf{v}_{01}$.

The last step in the calculational procedure is to transfer ("move") the vector $\mathbf{v}_1$ parallel to itself (preserving both its magnitude and its direction) from position $\mathbf{r}_0$ to position $\mathbf{r}_1$. We have completed our task: we have derived $\mathbf{r}_1$, $\mathbf{v}_1$ from $\mathbf{r}_0$, $\mathbf{v}_0$. We can repeat this algorithm: From $F_1 = -GMm/r_1^2$ derive $\Delta\mathbf{v}_{12}$, and then construct $\mathbf{v}_2 = \mathbf{v}_1 + \Delta\mathbf{v}_{12}$, plotting $\mathbf{r}_2$, $\mathbf{v}_2$, at time t_2, etc. *Thereby we trace out the orbit or trajectory of the body* moving under the gravitational force of the sun, without further recourse to observational data. After performing the calculations, we ask, does the prediction match future observation? For example, does the planet Mars or Halley's comet in fact appear in the night sky when and where we predicted it would? Within the limits of observational precision, yes, it does.[32]

31. In present-day notation, $\mathbf{F}(t_0) = -\mathbf{r}_0 GMm/r_0^3$. For a discussion of Newton's own distinctive kinematic-infinitesimal geometry in relation to the analytic-algebraic mathematics that quickly followed see my "The Exemplary Career of Newton's Mathematics," *The St. John's Review* 44, no. 1 (1997): 73–93. The differences between the two are not relevant for present purposes.

32. Thus our calculations have something to do with what is going on in the heavens. But

FIG. 2



In this account of the concept of *trajectory*, I have suppressed technical details (energy and angular-momentum considerations, non-linearity, the many-body problem, and calculus) involved in actual trajectory analysis in order to display what is essential to the kind of thinking characteristic of classical physics. In the following, I bring out two features of this Newtonian account that are not explicitly discussed by de Broglie, and that are of great importance from the side of the philosophy of nature, namely, (1) species-neutrality and (2) physico-mathematical secularism. With a view to the latter, let us look at the last step of the above procedure (in figure 2): the velocity vector (defined by its magnitude, i.e., speed, and its direction) can be detached from one position, $\mathbf{r}_0$ (where we performed the vector sum, $\mathbf{v}_0 + \Delta\mathbf{v}_{01}$), and "moved" on our plotting paper (always parallel to itself to keep it the same vector) to another position, $\mathbf{r}_1$. This is vivid testimony to our ability to conceive "motion apart from things" and our possession of an ingenious art of solving certain problems of local motion and force. Let us note, however, that this physico-mathematical art knows neither of Aristotle's act and potency, nor of Heisenberg's uncertainty principle.

"the heaven, even the heavens are the Lord's" (Ps 115:16). Have we (following Newton) entered into the mind of God? Is this belief the inspiration for the remarkable claims that we see in the following?

PART 3—SPECIES-NEUTRALITY: A NEW TYPE OF RELATION BETWEEN SENSIBLE AND INTELLIGIBLE

It is important to appreciate the remarkable character of Newton's law of gravitational force. In his own words, "there is a power of gravity pertaining to all bodies, proportional to the several quantities of matter which they contain … [and] inversely as the square of the distance."[33] Normally, the way a body moves or behaves is intimately related to what kind or species of body it is, as known through ordinary sense perception. Pigs do not fly, sparrows do not oink. Accordingly, the way in which two bodies interact depends on what kind each is. A dog and a cat interact in a certain way (the dog approaches to a critical distance at which the cat scratches its nose, then it rapidly recedes), a cat and a mouse in another (the cat approaches, the mouse recedes, but maybe not fast enough, and it gets caught and suffers a painful death). The traditional (Aristotelian and scholastic) name for the essential relation between the source of activity (form) and the supporting structure (matter) in a body is hylomorphism. It is reflected in assertions like the following in Aristotle: "Matter is among the relative things: for a different form, a different matter." "All things that change have matter, but there is distinct matter in distinct things."[34] Quite generally, in Aristotle's science of nature, the intelligible principles—natural form and common sensible matter—that account for the behavior of the sensed particulars (a cat, a tree, a stone) are species-typical or species-specific. Form, matter, and privation are indeed universal principles of natural things, but they are predicated of the different kinds (e.g., plants, animals, human beings) analogically, not univocally: "We cannot say that all things have the same elements and principles, except by analogy, just as if one were to say that there are three principles: form, privation, and matter. But each of these is different as it concerns each class of things."[35]

Newton's law of gravitation does not exemplify the Aristotelian, species-specific type of relation between sensible and intelligible.[36] The

33. Newton, *Mathematical Principles of Natural Philosophy*, trans. Andrew Motte and Florian Cajori (New York: Greenwood Press, 1962), Book III, Prop. 7 and Cor. 2.

34. *Physics* II.2.194b9 (53), *Metaphysics* XII.2.1069b25 (198); see also *Physics* VIII.1.251a12–13.

35. *Metaphysics* XII.4.1070b17–20 (200), translation slightly modified. The term "body" is thus, for Aristotle, not univocal, as it is for us, but analogical.

36. The general idea of laws of nature as species-neutral principles was pioneered by Bacon; see *New Organon*, I.51, I.66, I.88, II.3, II.35 (53, 63, 86, 122, 186).

equation "$F = -GMm/r^2$" expresses an intelligible principle of local motion in nature that is indifferent or neutral to the kind, size, shape, internal structure, and function of the two interacting bodies. Newton's gravitational law is thus species-neutral, as are the terms (mass, distance) of which it is composed. For all bodies—celestial and terrestrial, natural and artificial, living and nonliving—possess mass and relative position, also velocity, acceleration, momentum, kinetic energy, etc.[37] All the algebraic terms of classical mechanics are species-neutral. They are understood univocally, not, like form and matter, analogically. The algebraicization of science thus entails the homogenization of our thoughts and therewith a certain detachment of the resulting concepts from things. Klein describes this in an important passage of *Greek Mathematical Thought and the Origin of Algebra*:

> [In the] "new" science … nothing but the internal connection of all the concepts, their mutual relatedness, their subordination to the total edifice of science, determines for each of them a *univocal* [*eindeutig*] sense and makes accessible to the understanding their only relevant, specifically scientific content.… Thus every one of the newly obtained concepts [e.g., quantity, body, mass, motion, velocity, acceleration, momentum, force, work, energy] is determined by *reflection on the total context of that concept*. Every concept of the "new" science belongs to a new conceptual dimension. The special intentionality of each such concept is no longer a problem: it is indifferently the same for all concepts; it is a medium beyond reflection [*sie ist das allgemeine, von der Reflexion nicht mehr erreichte Medium*], in which the development of the scientific world takes place.[38]

Klein speaks here of the new concepts as being determined by reflection on their "internal connection . . . their mutual relatedness," namely, according to those axioms and operations of algebra, then according to the physical dimensions that they bear (e.g., mass, length, time, charge), and finally—an external connection—according to the operations of

37. Spinoza expresses succinctly the sense of the common (species-neutral) properties of what will become classical physics: "That which is common to all [bodies] … and which is equally in a part and in the whole [e.g., Cartesian extension, Newtonian mass], does not constitute the essence [the Aristotelian natural form; *Metaphysics* 1030a12] of any particular thing.… Those things which are common to all … cannot be conceived except adequately." *Ethics*, II.37, 38 (87, translation slightly modified).

38. Klein, *Greek Mathematical Thought*, 120–21.

measurement (e.g., our handling of scales, meter sticks, clocks) whereby they can be specified as, for example, 110 kilograms, 330 meters, 33 seconds. This is a new conceptual medium beyond the old (ancient and medieval) type of reflection on the "special intentionality" of each word. Consider, for example, the word "quantity," Greek *to poson*, Latin *quantum*, thus, more accurately (because more concretely), "the quantified," which expression points beyond itself and outside our thought to the quantified things. This ("quantified," *poson*, *quantum*) cannot be said univocally of the two kinds, discrete number, *arithmos*, and continuous magnitude, *megethos*, but only analogically. These two kinds of the quantified fall in one and the same category of being because they are both divisible into parts in such a way that both (*arithmos* and *megethos*) accept the equal and unequal (greater than, less than); but, whereas magnitude is infinitely divisible into parts having boundaries that can touch, number is finitely divisible into indivisibles—the units—lacking boundaries that can touch.[39]

This old type of reflection, on the ways "our thought, and also our words, signify or intend their [different kinds of] objects" would preclude the homogenization of the heterogeneity of discrete and continuous into the one univocal concept of the arithmetical continuum, or the real number system.[40] Similarly, for the old versus the new reflection on the words "body" (*soma*) and "motion" (*kinêsis*), and so forth, in natural science.[41]

We have gone from the species-neutrality of classical physics, to the univocalization of analogies, and homogenization of heterogeneities, in both mathematics and physics. We thus return to an important result first noted in our reading of de Broglie: mind's detachment from the world through the algebraicization of thinking.

A further implication derivable from Newton's gravitational law is the use of points to model physical processes. Since all bodies and their

39. Aristotle, *Metaphysics* V.13, *Categories* VI. The even and the odd belong to number, but not to magnitude. A square magnitude can always be divided into two equal square magnitudes, but a square number can never be divided into two equal square numbers.

40. Klein, *Greek Mathematical Thought*, 118.

41. In "The World of Physics and the 'Natural' World," Klein seems to say that in fact the late scholastics had drifted into this new conceptual medium without knowing it and thus without knowing how to use it to maximum effect, which involves turning the mind in a new direction and questioning in a new way. *Lectures and Essays*, 3, 6–7.

parts, regardless of kind, size, internal structure, and function, have mass and attract each other pairwise according to $F = -GMm/r^2$, as long as the spatially extended bodies in a gravitational system do not bump into each other, they can be represented mathematically as unextended points: *mass* points. Recall that, according to Aristotle, in *Physics* VI.4, "every changing thing is necessarily divisible."[42] An unextended, indivisible point, therefore, could be moved only *per accidens* by being in or on something that is itself *per se* movable (movable through itself), which, therefore, has an interior and is necessarily extended. A physical-material point is, on this account, a contradiction in terms. But for the solution of many problems in the physics of gravitational systems, this piece of natural philosophy is not needed. In these cases, the spatial extendedness, the coherent shapes and sizes of bodies do not matter. And, in other cases (e.g., the study of gases, liquids, and solid bodies), where they do matter, perhaps they can be accounted for by reasoning that is in some way analogous to the treatment of the solar system. Indeed Newton says, "Nature is exceedingly simple and conformable to herself. Whatever reasoning holds for greater motions [e.g., of the solar system] should hold for lesser ones [of the particles of bodies] as well."[43] This postulate—of the multi-level identity of nature whereby there is no natural scale of size—makes possible Newton's remarkable generalization of his gravitational theory.[44]

42. *Physics* VI.4.234b10 (153).

43. "Unpublished Conclusion of the *Principia*," in A. R. and M. B. Hall, *Unpublished Scientific Papers of Isaac Newton*, 333. A more compressed formulation (that omits explicit reference to greater and lesser motions) is given in Rule III of the Rules of Reasoning in Philosophy: "nor are we to recede from the analogy of Nature, which is wont to be simple, and always consonant to itself." Since we find that sensible bodies possess hardness, impenetrability, mobility, and inertia, "we conclude the least particles of all bodies to be also all extended, and hard, and impenetrable, and movable, and endowed with their proper inertia. And this is the foundation of all philosophy. . . . Lastly . . . we must . . . universally allow that all bodies whatsoever [both big visible planets and tiny invisible particles] are endowed with a principle of mutual gravitation." *Mathematical Principles*, Book III, 398–400. Finally, *Opticks*, Query 31 (New York: Dover, 1952), 375–406, esp. 394 and 397. For the Cartesian version of "the analogy of Nature," see Descartes, *Principles of Philosophy*, 4.20: "No one who uses his reason will, I think, deny the advantage of using what happens in large bodies, as perceived by our senses, as a model for our ideas about what happens in tiny bodies which elude our senses merely because of their small size. This is much better than explaining matters by inventing all sorts of strange objects which have no resemblance to what is perceived by the senses <such as 'prime matter,' 'substantial forms' . . . >. " (CSM I 287).

44. The phrase "multi-level identity of nature" is Laurens Laudan's. See Laudan, "The

PART 4—REDUCTIONIST GENERALIZATION: NEWTON'S UNIVERSAL FORCES-AND-PARTICLES MODEL

The species-neutrality of the law of gravitational force facilitates Newton's remarkable generalization from his particular gravitational theory to the universal forces-and-particles model, a mental image of everything physical in the whole universe. Newton proposes this in the preface to the *Principia*:

> I derive from the celestial phenomena the forces of gravity with which bodies tend to the sun and the several planets. Then from these forces … I deduce the motions of the planets, the comets, the moon, and the sea. I wish we could derive the rest of the phenomena of Nature by the same kind of reasoning from mechanical principles, for I am induced by many reasons to suspect that they may all [!] depend upon certain forces by which the particles of bodies, by some causes hitherto unknown, are either mutually impelled towards one another, and cohere … or are repelled and recede.… These forces being unknown, philosophers have hitherto attempted the search of Nature in vain.[45]

Here, every body is like a solar system writ small. This is a grand analogy, but not of ancient or medieval type (conveying ontological sameness and difference). Rather, it is conceptually homogeneous, like "big circles are analogous to small circles." All the sensible, composite bodies are mentally conceived as clouds of subsensible particles, which move in space on in-principle calculable trajectories. It is assumed here that the intelligible principles of natural phenomena will, like the gravitational force law, be expressible in species-neutral terms like mass, and the spatial relations of particles, point-like centers of attraction and repulsion. The discovery of electric charge and Coulomb's law of electrical attraction and repulsion, similar in its algebraic form to Newton's, gave the Newtonian program great impetus. Thus in 1847 Helmholtz proclaimed the goal of physical science as the complete intellectual penetration of nature by human mind:

Clock Metaphor and Probabilism: The Impact of Descartes on English Methodological Thought, 1650–65," *Annals of Science* 22, no. 2 (June 1966): 73–104, esp. 91–92. The postulate of the multi-level identity of nature clashes with ordinary experience, is therefore incompatible with Aristotle's physics, and is refuted by quantum physics.

45. Newton, *Mathematical Principles*, 1686 Preface, xviii.

> Natural phenomena are to be related to the motions of matter possessing unchanging forces of motion, which forces depend only on spatial relations. . . . The force, however, which two whole masses exert on each other must be resolved into the forces of all their parts on one another; thereby mechanics goes back to the forces of material points, that is, to the points of space filled with matter. . . . Finally, then, the task of the physical natural sciences is specified thus: to reduce natural phenomena to unchanging attractive and repulsive forces, whose strength depends on the distance. The realizability of this task is, at the same time, the condition of *the complete comprehensibility of nature*.[46]

De Broglie's general description is here, in the words of Newton and Helmholtz, exemplified for classical mechanics. According to this world-conception, all the properties and activities of all the wholes in nature are derivable from, or somehow entailed by the local motions and quantitative properties of their constituent particles. Thus mass, charge, associated force laws, particle position, velocity, momentum, energy, associated densities, distributions, and flows are taken to be the terms adequate for the explanation of all natural phenomena.

What does the forces-and-particles model of the universe imply, according to its own inner logic, about nature as we prescientifically encounter it, specifically, as articulated into visible kinds or species, for example, "lion, eagle, rose, gold, and the like"?[47] The following paradoxical implications clash with our ordinary sense-perception-based experience of, and belief about natural things, both living (a lion, an eagle, a rose) and nonliving (gold), but especially about living things.

Wholes are reducible to sufficiently simple parts or particles. Thus there are no holistic systems—compounds that are irreducible to their parts—in nature. Wholes that, unlike artifacts, strike us as irreducible to parts (they seem to be more than aggregates) are the ones we call "alive," because of their intrinsic unity (if we try to pull an animal apart it bites and scratches, works hard to keep itself together) and characteristic stability of their kind (cats have kittens, dogs have puppies). Biological

46. "On the Conservation of Force," *Wissenschaftliche Abhandlungen* (Leipzig, 1882), 1:15–16; author's translation (emphasis added).

47. Bacon, *New Organon* II.17 (152). Bacon is well aware of the dualism of subsensible particles and laws, on the one hand, and sense-perceptible compounds, on the other—a dualism of "the homogeneity of laws and the heterogeneity of kinds" in Thomas Prufer's apt phrase—and thus of the need to explain how the former give rise to the later. But no such explanation is provided in the *New Organon*.

phenomena fuel Aristotle's account of form as a holistic principle, discussed in part 6, below.

"Sufficiently simple" in the first sentence of the preceding paragraph means *with respect to a force law*; that is, in the quantitative division of bodies, we come to a level of smallness such that the particles (bearing properties like mass and electrical charge) obey one or more mathematically expressible force laws enabling the calculation of trajectories from initial data, on the Newtonian model as described in part 2, above. It is the assumed availability of such laws ("certain forces by which the particles of bodies [interact]") and resulting equations of motion that grounds this type of reductionism, not the existence of ultimate or metaphysically first particles, like the unbreakable atoms of Democritus. Indeed, a particle that is taken as an indivisible whole at one stage of theory can be further reduced to subparticles at a later stage if corresponding forces and force laws are discovered. In this sense, the first principles of nature are laws, not beings.[48] I resume my list of paradoxical implications.

There is no true genesis, no real novelty in nature, no substantial change, because the only real motion in nature is the local motion of particles under unchanging laws of force. Sense-perceptible qualities and their changes, for example, alteration, must then be intramental representations arising through some sort of psychophysical association with extramental particle motions.

There are no privileged moments or states, no ends in nature. A configuration of particles moves under the equations of motion as determined by initial data (e.g., positions and velocities) at any given time.[49] There is no "room" in the basic logic of the trajectory calculation for a future state, for example, health (something we all experience to be a natural end), to be a cause of present motion, for example, healing, as discussed in part 7, below. Furthermore, the equations of particle

48. See Kennington, *Modern Origins*, 33–56. The unity of nature would then be grounded in the most general laws, Bacon's *summa lex* or *magna forma* (*Advancement of Learning*, *Works* 3.265, *New Organon*, II.26, II.52), or, in contemporary parlance, the Theory of Everything. But practical success—prediction and control of natural processes and the creation of remarkable technologies—does not require knowledge of the ultimate laws.

49. "We ought then to regard the present state of the universe as the effect of its anterior state and as the cause of the one which is to follow.... The regularity which astronomy shows us in the movements of the comets doubtless exists in all phenomena. ... The only difference is that which comes from our ignorance" of the present positions and velocities of all the particles and the forces by which they interact. Laplace, *Probabilities*, 6.

motion are invariant under time reversal (replacing t and v with –t and –v), which implies that the arrow of time conspicuously evident to us all has no basis in (what are here conceived to be) the fundamental principles of nature.[50]

Finally, nature is completely malleable: one kind of body can be transformed into another. In Newton's own words, "every body can be transformed into another, of whatever kind, and all the intermediary degrees of qualities can be successively induced in it."[51] Newton is not simply saying here, in Hypothesis III, what ancient materialists said, that is, that nature is in flux, that any body will at some time go back to its elements, which will then enter into the composition of some other body.[52] Newton's remarkable assertion of *transformism* follows the inspiration of Bacon: "On a given body to generate and superinduce a new nature or new natures is the work and aim of human power."[53] The transformation of bodies takes place according to human will.

PART 5—TRANSFORMISM: AN IMMODERATE DISPOSITION TOWARD NATURE

How does transformism follow from the universal forces-and-particles model? I believe it follows quite simply, and present a deduction of it that Newton never explicitly presented but which is faithful to the logic of the forces-and-particles model.

If the intelligible principles are species-neutral, then the heterogeneity of species, so evident to our senses, for example, eagle, rose, gold, must not result *per se* from those intelligible principles, but must rather be, somehow, accidental. That is: the sensible species are not the effects of

50. The problem of irreversibility in the foundations of physics persists to this day: "At the level of particles [more precisely, wavicles], things can happen in reverse, because particles obey time-symmetric laws of mechanics. But then why does matter, which is made up of these building blocks, behave irreversibly?" Oliver Penrose, "An asymmetric world," *Nature* 438 (December 15, 2005): 919.

51. Newton, *Principia*, First Edition, Hypothesis III.

52. Nor is Newton's assertion an echo of Aristotle's doctrine of the transmutation of the elements through the open-ended potency of primary matter (under the four elemental forms, earth, air, fire, water) and the agency of the sun. As discussed in Part 6, on Aristotle, the higher natural forms, e.g., squirrel-form, are *per se* unchangeable and strongly limit the malleability of composite natural substances.

53. Bacon, *New Organon* II.1 (121).

causes aimed *per se* at those effects (like Aristotelian form and matter), and so their heterogeneity is not rooted in the essential nature of things, and, furthermore, might not be a barrier to our operation.

For example: in light of Newton's law of universal gravitation, we learn that there is no essential difference or heterogeneity of celestial and terrestrial bodies. In fact, in light of Newton's laws of motion and gravitational force, humanly controlled space flight is discovered to be possible, to be within our power, so that we can choose to do it. By launching a body from earth, namely, a rocket, with enough velocity, engineers can make it go up into the heavens and, metaphorically speaking, transform a terrestrial body into a celestial body. This is in fact shown at the end of Newton's *Principia*.[54] Moreover, by means of rocket stages and appropriate midcourse burns, following the same physics, the engineers can control the trajectory of the rocket and make it go, to a significant extent, wherever we desire. Now this is a wonderful but particular piece of physics and engineering; we have not mastered all physical phenomena but only one type, those of gravitational systems. But look what happens to this particular case when, in our imagination, we apply to it Newton's universalizing analogy: "Whatever reasoning holds for greater motions [the earth, the rocket, the solar system] should hold for lesser ones [the tiny particles of a body] as well."[55] Therefore, one imagines, just as engineers can shape the trajectory of a rocket in the solar system, so also they should be able in the future to control the trajectories of the minute particles in a cloud that we have traditionally called, say, "a cat" and thereby shape that cloud into one that we have traditionally called "a dog." This may be *practically* difficult today, due to the smallness of the particles and their large number, but there is nothing in the *nature of things* to prevent this—nothing about material being and its principles, or the process of sense perception, or, as de Broglie describes, the use of experimental apparatus that might limit our knowledge and control of the phenomena.[56] By this logic, then, transformism is demonstrated.

54. *Principia* (Motte-Cajori, 551).

55. "Unpublished Conclusion of the *Principia*," in A. R. and M. B. Hall, *Unpublished Scientific Papers of Isaac Newton*, 333.

56. It is a basic feature of quantum phenomena that the act of measurement disturbs and alters the entities under study in an essential (thus quantitatively irreducible) way. Whereas a planet is what it is independently of the reflected light whereby we observe it, the same cannot be said of an electron. "The impossibility of a closer analysis of the reactions between the

Newton's Hypothesis III of the first edition of the *Principia* is extreme. It clearly implies the ability to transform a dead body into a living one, and would thereby bring bodily immortality within human power. It is not part of the warranted (mathematical and hypothetico-deductive physical) reasoning of the *Principia*, and Newton removed it from the second and later editions. But the outlook it expresses is not unique to Newton; it is a disposition characteristic of early modern philosophy and the accompanying new science of nature.[57] Here are Bacon (also above), Descartes, and Spinoza on the same theme:

Therefore a separation and solution of bodies must be effected, not by fire indeed, but by reasoning and true induction, with experiments to aid [...]. In a word, we must pass from Vulcan to Minerva if we intend to bring to light the true textures and configurations of bodies on which all the occult and, as they are called, specific properties and virtues in things depend, and from which, too, the rule of every powerful alteration and transformation is derived.[58]

I took pains to make everything belonging to the nature of fire very clearly understandable.... Thus I made clear how it is formed and fueled, how sometimes it possesses only heat without light, and sometimes light without heat; how it can produce different colors and various other qualities in different bodies; how

particle and the measuring instrument is indeed no peculiarity of the experimental procedure described, but is rather an essential property of any arrangement suited to the study of the phenomena of the type concerned, where we have to do with a feature of *individuality* [the entity studied and the instrument that studies it form an indivisible whole] completely foreign to classical physics." Neils Bohr, "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?," *Physical Review* 48 (October 15, 1935): 696–702, here 697. "Phenomena in atomic physics possess a new property of wholeness, in that they cannot be dissected into part-phenomena without changing the entire phenomenon substantially every time [a measurement] is attempted." Wolfgang Pauli, "Naturwissenschaftliche und Erkenntnistheoretische Aspekte der Ideen vom Unbewussten," *Dialectica* 8 (1954): 285. In terms of classical physics itself, there is a fundamental problem in my demonstration of transformism: the ability to effect extraordinarily delicate manipulations of minute particles that I have attributed to future engineers (they can transform a cat into a dog) would enable them as well to perform the operations of Maxwell's demon whereby heat is made to flow from cold to hot without expending energy, in violation of the second law of thermodynamics, of which Newton did not know. See James Clark Maxwell, *Theory of Heat* (New York: Dover, 2001), 328–29.

57. As noted in the Preface and Introduction, the belief in and desire for the unlimited mastery of nature abides. It is a disposition incompatible with any understanding of nature as normative: "Everything is in principle open to intervention; because all is alterable, nothing is deemed either respectably natural or unwelcomely unnatural" (Leon Kass, *Toward a More Natural Science*, 11).

58. Bacon, *New Organon*, II.7 (128).

it melts some bodies and hardens others; how it can consume almost all bodies, or turn them into ashes and smoke; and finally how it can, by the mere violence of its action, form glass from these ashes—something I took particular pleasure in describing since it seems to me as wonderful a transmutation as any that takes place in nature.[59]

Nothing comes to pass in nature, which can be set down to a flaw therein [cf. Aristotle, *Physics* 199b4]; for nature is always the same, and everywhere one and the same in her efficacy and power of action; that is, nature's laws and rules, whereby all things come to pass and change from one form to another, are everywhere and always the same; so that there should be one and the same understanding of the nature of all things whatsoever, namely, through nature's universal laws and rules.[60]

Not form, but transformation is the point. Such transformation is the focus of will, reason, and imagination in each of these passages.

In developing the theme of transformism, I began with the species-neutrality of the law of gravitation. I did not end there, for Newton's forces and particles model is not only species-neutral but, as clearly noted, reductionist: wholes are reducible to parts or particles; parts are prior to wholes, ontologically and thus epistemologically. The absolute priority of parts to wholes is necessary for Newton's radical thesis of universal transformism. It is because the (sufficiently simple) parts of one species of whole, such as a cat, are indifferent to, or unmodified by their membership in that whole that they can be removed from it and put into some other kind of whole, such as a dog, or used to compose a new kind of whole altogether. If this were not the case—that is, if the parts or particles of certain kinds of wholes, and the force laws by which they interact, were somehow affected or modified or limited in their being by their membership in that kind of whole—then it would not be possible to use those particles as universal building blocks that could be disassembled and reaggregated in the transformation and synthesis of other kinds of wholes.

Let us now reverse course to take a further look at the Aristotelian alternative, an understanding of nature that is species-specific and holistic, and accordingly less technologically potent.

59. Descartes, *Discourse on the Method*, Part 5 (AT VI 44–45, CSM I 133).
60. Spinoza, *Ethics* III, Introduction (129).

Part 6—On Aristotle's Science of Nature: Form Prior to Matter, Wholes Prior to Parts

We have two distinctions: (1) between species-neutral and species-specific principles of nature and science; (2) between reductionist and holist whole-part relations. I have used Newton's law of gravitation and the forces-and-particles model to exemplify the first terms in these distinctions (species-neutral, reductionist). Aristotle's doctrine of natural form (*morphê*, *eidos*) and natural substance (*ousia*) exemplify the second terms (species-specific, holist).

Physics II.1 provides the argument for form as principle and cause of change and stability in a natural substance: "The things existing by nature [not by art] all appear to have *within themselves* a principle of motion and rest. … [So] nature is a certain principle and cause of being moved and of coming to rest *in* that to which it belongs, primarily and essentially and not accidentally."[61] Nature is both form and matter, and "the form is nature more than the matter."[62] The complicated adverbial phrase, "primarily and essentially and not accidentally," means (among other things) that natural form is internal to the moved thing, such as an animal, in a way that cannot be fully derived from, or completely reduced to its material parts: "[Natural] things will be neither without matter nor determined by their matter."[63] Form is thus a holistic principle; the parts of a naturally informed compound are what they are and act as they do only in terms of the whole they compose. If separated by dissection from the whole, they cease to be what they were:

> The whole must of necessity be prior to the part; for if the whole [body] is destroyed there will not be a foot or a hand, except in the sense that the term is similar (as when one speaks of a hand made of stone).… For it is not in any manner whatsoever that a hand is a part of the body, but only when it can perform its function, so it must be alive; if not, it is not a part.[64]

61. "*Ta men gar phusei onta panta phainetai echonta* en heautois *archên kinêseôs kai staseôs.* […] *hôs ousês tês phuseôs archês tinos kai aitias tou kinesthai kai êremein* en *hô huparchei prôtôs kath'auto kai mê kata sumbebêkos.*" *Physics* II.1.192b14–15, 21–23 (25); emphasis added. The *appearance* of having an irreducibly internal source of motion is accepted by Aristotle as reliable.

62. *Physics* II.1.193b7 (27, translation slightly modified).

63. *Physics* II.2.194a14–15 (52, translation slightly modified).

64. *Politics* I.2.1253a20–22, trans. Carnes Lord, *Aristotle's* Politics (Chicago: University of

Therefore, one whole informed material substance, such as a squirrel, cannot be adequately understood in terms of its parts. Rather, since the whole is ontologically prior to the parts, the parts cede some of their being to the unifying authority of the form. Thus, for Aristotle, the parts of a natural substance exist only potentially (*dunamei*) in the whole; they are not fully actual in the whole: "And the continuous and limited is said to be a whole when it is some one thing consisting of many constituents, most of all when these exist potentially, but if not, actually.... Whenever the parts [of an animal] are one and continuous by nature ... they will exist potentially."[65] In fundamental contrast, the parts of an artifact, such as a clock, are what they are independently of the whole; they are fully actual, and thus in their being are neutral or indifferent to the whole. Therefore, a watch, which lacks the substantial unity of that squirrel, can be taken apart, its parts can be analyzed in isolation from each other, and then both the parts and our knowledge of the parts can be reaggregated in the whole, or even rearranged in a new and different "species" of whole.[66] Here, the parts are prior, ontologically and epistemologically, to the whole. The clock—in which we can literally "see all the wheels functioning"—is thus a paradigm for the reductionist and anti-Aristotelian conception of the whole-part relation. In general,

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Chicago Press, 2013), 4; *Metaphysics* VII.11.1036b31–32 (126); see also VII.10.1035b23–25. Hegel's (post-Newtonian) formulation of the whole-part relation in a living thing accords with that of Aristotle: "The notion of the whole is to contain parts: but if the whole is taken and made what its notion implies, i.e., if it is divided, it at once ceases to be a whole. Things there are, no doubt, which correspond to this relation: but for that very reason they are low existences [...]. The relation of whole and parts comes very easy to reflective understanding; and for that reason it often satisfies when the question really turns on profounder ties. The limbs and organs, for instance, of an organic body are not merely parts of it: it is only in their unity that they are what they are, and they are unquestionably affected by that unity, as they also in turn affect it. These limbs and organs become [mere] parts, only when they pass under the hands of the anatomist, whose occupation, be it remembered, is not with the living body but with the corpse. Not that such analysis is illegitimate: we only mean that the external and mechanical relation of whole and parts is not sufficient for us, if we want to study organic life in its truth." Hegel, *Encyclopedia Logic*, §135, note; see also §38, note.

65. *Metaphysics* V.26.1023b33–35, VII.16.1040b14–17 (98, 133, translation slightly modified); see also *De Anima* II.2.414a20–28.

66. Within Newtonian physics, the reductionist or mechanical relation of whole and part (parts unaffected by the whole) is implicit in the parallelogram rule for composition of forces, Corollary II of Newton's *Principia*, whereby the conjoint action of the particles composing a compound body is assumed to be the sum of the actions (forces) of each particle taken separately. See my "Wholes, Parts, and Laws of Motion," *Nature and System* 6 (1984): 195–215, and "Animals versus the Laws of Inertia," *Review of Metaphysics* 46 (1992): 29–61.

reductionism is a sufficient but not necessary condition of species neutrality. For example, Darwin's biology is species-neutral but not reductionist in the strong (material) sense used here.[67] Conversely, a species-specific account, like Aristotle's doctrine of natural substance, is, to a significant extent, necessarily holist.

Aristotle's doctrine of parts being potentially in the whole is admittedly obscure. We can get a sense of it only by consideration of his examples, which are almost all biological.[68] I argue in part 7, below, that it has relevance as well for the quantum physics of atoms and molecules. In any case, despite the obscurities, it is fair to say that, in the Aristotelian account, the relativity and subordination of the being of the parts of a natural compound to its unifying form necessarily implies limits to the alteration, manipulation, and transformation of bodies by separation and recombination of their parts. To separate the part from the whole is not only to mutilate the whole but, unlike the gear removed from the clock, to alter the part so that it cannot "perform its function." Within this understanding, then, the form is not only a source but also a limit to our knowledge and control of nature: "Limit means … the substance of each thing, and the essence [*to ti ên einai*] of each thing, for this is a limit of knowledge, and if of knowledge then of the thing also.… Essence will belong to nothing which is not a species [*eidos*] of a genus, but only to a species of a genus.… By form [*eidos*] I mean the essence of each thing."[69] In particular, our ability to know what is going on inside a natural substance is limited. Aquinas expresses the Aristotelian teaching succinctly: "natural forms are not *per se* subject to motion […] they are, moreover, the perfections of mutable things."[70]

Aristotle's species-specific, form-limited holism stands at the oppo-

67. Random variation and natural selection are governing principles common to all living species, but (to my knowledge) Darwin never claimed that an animal is nothing but an aggregate of particles. On the species-neutrality of Darwinian biology, see my "Darwinian Natural Right?," *Interpretation* 27, no. 2 (2000): 129–60; Larry Arnhart, "Defending Darwinian Natural Right," *Interpretation* 27, no. 3 (2000): 263–77; and my "Reply to Arnhart," *Interpretation* 28, no. 1 (2000): 35–43.

68. The exception is parts (intervals) of a line potentially present in the line, *Physics* VIII.8.263a28.

69. Aristotle, *Metaphysics* V.17.1022a5–10, VII.4.1030a12–13, VII.7.1032b2 (93, 112, 117).

70. Aquinas, *In de trinitate*, q. 5, a. 2, ad 6; translated by Armand Maurer in Thomas Aquinas, *The Division and Methods of the Sciences* (Toronto: Pontifical Institute of Medieval Studies, 1986), 31.

site extreme to Newtonian species-neutral and reductionist transformism. Is there a reasonable mean? The reality of laws of nature and the particular successes of reductionist explanation make clear that nature is less Aristotelian than Aristotle thought. But classical physics suffered its own great embarrassment on the problem of the stability of matter, the problem of how to account for the specific properties of atoms and molecules, that is, of the chemical species. This is discussed in the following section. Preparatory to that discussion, consider the following by J. S. Mill in 1843, "On the Composition of Causes":

The preceding discussions have rendered us familiar with the case in which several agents, or causes, concur as conditions to the production of an effect. [...] Suppose, then, that two different agents, operating jointly, are followed, under a given set of collateral conditions, by a given effect.... Now, if we happen to know what would be the effect of each cause when acting separately from the other, we are often able to arrive deductively, or *a priori*, at a correct prediction of what will arise from their conjunct agency. To render this possible, it is only necessary that the same law which expresses the effect of each cause acting by itself shall also correctly express that part due to that cause of the effect which follows from the two together. This condition is realized in the extensive and important class of phenomena commonly called mechanical [which are based on] the principle of the [parallelogram rule for] Composition of [Newtonian] Forces: and, in imitation of that well chosen expression, I shall give the name of the Composition of Causes to the principle which is exemplified in all cases in which the joint effect of several causes is identical with the sum of their separate effects.

This principle, however, by no means prevails in all departments of the field of nature. The chemical combination of two substances produces, as is well known, a third substance with properties different from those of either of the two substances separately, or of both of them taken together. Not a trace of the properties of hydrogen or of oxygen is observable in those of their compound, water. ... This explains why mechanics is a deductive or demonstrative science, and chemistry is not....

This difference between the case in which the joint effect of causes is the sum of their separate effects, and the case in which it is heterogeneous to them; between laws which work together without alteration, and laws which, when called upon to work together, cease and give place to others; is one of the fundamental distinctions in nature.[71]

71. John Stuart Mill, *A System of Logic*, Book III, Chapter VI (London: Longmans, Green

Chemistry can become “a deductive or demonstrative science” only on grounds of a new type of physics in which the principle of the composition of causes does not hold.

PART 7—STABILITY OF MATTER: RADICAL FAILURE OF CLASSICAL PHYSICS

The species-neutral reductionism of classical physics is so radical that it directly contributes to one of the great scientific revolutions of the twentieth century, quantum physics. Rutherford’s analysis of scattering experiments in 1911 led to the nuclear or “planetary” model of the atom: a dense, positively charged nucleus surrounded by smaller electrons in a much larger environing space. It seems, at first (conceptual) glance, like a tiny solar system (hence the name “planetary”) and thus like a vindication of Newton’s universal forces and particles model.[72] But will the postulate of the multi-level identity of nature hold, that is, will “the same kind of reasoning from mechanical principles” work on the atomic scale of size as it does on the astronomical and terrestrial-engineering scale? It will not. The classical conception of the nuclear atom poses the problem of the stability of matter. It is a problem with two parts, internal and external, as follows.

The problem of internal atomic stability. Consider an isolated atom. Negatively charged electrons must be strongly attracted (by the Coulomb force) to the positively charged nucleus (containing protons and neutrons). What maintains the electrons in place around the nucleus—either at fixed positions or, if we conceive them to circulate like tiny planets, in orbits of fixed radii—against the electrical force that pulls them in to the nucleus? For, unlike an orbiting planet in Newtonian gravitational theory, an orbiting charged particle in classical electromagnetic theory must emit electromagnetic radiation, which depletes its kinetic energy;

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and Co., 1941), 242–44. See note 64 (above), on the composition of forces. Newton well appreciated the significance of the parallelogram rule: “the use of this Corollary [II] spreads far and wide, and by that diffuse extent the truth thereof is further confirmed. For on what has been said depends the whole doctrine of mechanics” (*Principia*, 17).

72. E. Rutherford, “The Scattering of α and β Particles by Matter and the Structure of the Atom,” *Philos. Mag.* 21 (April 1911): 669–88. The characteristic radius of the solar system is 10^{12} meters, of a hydrogen atom 10^{-10} meters. The assumption that the same principles should hold on both scales is extraordinary.

continuous acceleration along its classical trajectory entails continuous energy loss by the moving particle. What then happens?

The problem of external atomic stability. Consider the many atoms composing a liquid or a solid, atoms so closely packed that, unlike a gas, the materials they compose resist compression. What enables each atom to maintain its shape and integrity, and thus its specific characteristics, against the strong external disturbances (crunching against the other atoms or, if they are on the surface of the liquid or solid, being buffeted by light) to which it must be continually exposed?

Classical physical theory cannot provide answers to these questions. Classical theory makes the stability of atoms—wholes composed of nuclei and electrons—unintelligible, and therewith the properties of the chemical species composed of atoms. We prescientifically experience these properties all the time in, for example, the solidity of our bones and of the chair we are sitting on, and we scientifically observe them in sophisticated laboratory experiments, for example, the color spectra of light emitted from gases under electrical stimulation (a strong external disturbance). This is the world-historic failure of classical physical theory: it cannot account from its own first principles for the evident specificity of the material world.[73]

The failure of classical physics on the internal atomic stability problem is described in standard textbooks. There is no stable equilibrium configuration of static, electrically charged particles, a theorem originally attributed to Samuel Earnshaw and immediately derivable from Laplace's equation, $\nabla^2\varphi = 0$, for the electric potential in free space.[74] For electrons circulating around the nucleus, there is no stable configuration

73. To be sure, for about two centuries, classical physics solved all sorts of engineering problems in which the stable properties of liquids and solids were taken for granted and incorporated in the equations as boundary conditions or empirically determined constants. For example, water is incompressible and has a given viscosity, while cement is solid, unlike butter, and will contain the water in a swimming pool, whose surface will be horizontal in equilibrium in the earth's gravitational field. If disturbed, the water will propagate surface waves and eventually return to its stable equilibrium state with a flat surface. This does not, however, explain the respective characteristics of water and concrete in terms of their atomic and molecular constituents, or nuclei and electrons. For this, quantum physics is required.

74. See William Thomson and P. G. Tait, *Treatise on Natural Philosophy* (Oxford: Clarendon Press, 1857), 372–73; L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields* (Reading, Mass.: Addison-Wesley, 1962), 100; Feynman, *Lectures*, vol. 2, §5–2, especially "Stability of Atoms."

of orbits due to radiative energy loss, whereby the electron would spiral into the nucleus in a tiny fraction of a second.[75] In other words, executing the trajectory calculation of part 2, above, using Coulomb's law for the electrical force of the nucleus on an electron (of mass, me, and charge, e) beginning from initial position and velocity, ro, vo, just as we did using Newton's law for the gravitational force of the sun on a planet (of mass, m), leads, not to a spectacular success, but a radical failure. An electron in an atom does not behave according to the properties that define it classically—mass and charge—and that would determine classically its local motion in an electric field. Rather, the phenomena of atomic stability require that the electron be altered, modified, in some way limited by its membership in the whole atom (were this not the case, the atom could be explained through Mill's "Composition of Causes"). Is this not a distant echo of Aristotle's obscure doctrine of material parts potential in the informed whole?[76] As Wallace describes it:

> The organization or formal arrangement of these components [electrons, protons, and neutrons], and not the components themselves, makes [e.g.,] sodium be what it is.... None of the three components of the sodium atom acts simply as an electron, proton, or neutron, ... each functions instead as a part of sodium.... On its own, each electron would be indifferent to the particular energy state it might occupy within the atom; within the atom, according to the Pauli exclusion principle, each electron is assigned to a unique state occupied by no other.[77]

Aristotle's obscurity will be removed by the mathematical theory of quantum states, their superpositions and transformations, but that theory removes as well the central concepts of classical physics: particle

75. R. M. Eisberg, *Fundamentals of Modern Physics* (New York: John Wiley and Sons, 1961), 108–9; see also 366–69 on the great importance of the Pauli exclusion principle, namely, "in a multi-electron atom there can never be more than one electron in a given quantum state" (366); without this, "atoms, and therefore the entire universe, would be *radically different*" (368).

76. The notion of parts existing potentially in the whole is complemented by the doctrine of parts being virtually present in the whole: the electron retains its mass, m_e, and charge, q_e, in an atom (this is the unmodified presence), but the function of the electron is not given by the classical equations for a particle of mass, m_e, and charge, q_e, in the field of the nucleus; what the particle is when it is inside the atom is thus modified relative to its nature as classically conceived.

77. William A. Wallace, *The Modeling of Nature* (Washington, D.C.: The Catholic University of America Press, 1996), 46–47.

trajectory and field magnitude (spatio-temporally continuous and deterministic); they are not adequate to nature. Let us continue the story of the stability of matter. It remains to discuss the stability of atoms against external disturbances.

The failure of classical physics on the external stability problem is succinctly described by Niels Bohr as paraphrased by Werner Heisenberg in the following excerpt, which situates the difficulty precisely in the notion of deterministic particle trajectory:

> My starting point was not at all the idea that an atom is a small-scale planetary system and as such governed by the laws [like those] of astronomy. I never took things as literally as that. My starting point was rather the stability of matter, a pure miracle when considered from the standpoint of classical physics.
>
> By "stability" I mean that the same substances always have the same properties, that the same crystals recur, the same chemical compounds, etc. In other words, even after a host of changes due to external influences, an iron atom will always remain an iron atom, with exactly the same properties as before. This cannot be explained by the principles of classical mechanics, certainly not if the atom resembles a planetary system. Nature clearly has a tendency to produce certain forms . . . and to recreate these forms even when they are disturbed or destroyed. You may even think of biology: the stability of living organisms, the propagation of the most complicated forms which, after all, can exist only in their entirety. But in biology we are dealing with highly complex structures, subject to characteristic, temporary transformations of a kind that need not detain us here. Let us rather stick to the simpler forms we study in physics and chemistry. The existence of uniform substances, of solid bodies, depends on the stability of atoms; that is precisely why an electron tube filled with a certain gas will always emit light of the same color, a spectrum with exactly the same lines. All this, far from being self-evident, is quite inexplicable in terms of the basic principles of Newtonian physics, according to which all effects have precisely determined causes, *and according to which the present state of a phenomenon or process is fully determined by the one that immediately preceded it*. This fact used to disturb me a great deal when I first began to look into atomic physics.[78]

On grounds of classical theory, then, there is no way in which a future state could be a cause of present motion.[79] This makes unintelligible (or

78. Heisenberg, *Physics and Beyond*, trans. A. J. Pomerans (New York: Harper Torchbooks, 1972), 39; emphasis added.

79. This is an imprecise but didactically useful formulation of final causality.

a matter of extraordinary improbability, “a pure miracle”) health and healing (“the stability of living organisms”), and, more to the point, the *ground state* characteristic of a given species of atom, to which the atom returns by emission of specific frequencies of light after it is disturbed by an external influence.

We can see exactly what Bohr is getting at by means of the trajectory calculation, in part 2, above: imagine that a typical classical system, the solar system, suffers a strong external disturbance. Say a large comet or asteroid passes through the solar system, not colliding with any planets, but pulling them off of their previous orbits through its own gravitational force. Is there anything in the fundamental principles of Newtonian physics—the principles that we used to calculate a trajectory from given initial conditions—that would cause the planets to recover their previous orbits? The answer is, no, for the effect of the comet or asteroid is simply to “reset” the initial conditions, the positions and velocities of the planets, which then *fully* determine the future trajectories under the laws of motion and force. There is no room in this classical kind of reasoning for the solar system somehow to remember, as it were, its past configuration and get back to it.

The radical species-neutrality and reductionism of the classical world conception make nature completely indifferent to itself, thus without privileged states that are specific to the kind and self-reconstitutive—like the ground states of the atoms of the chemical elements. As elemental, a given kind of atom enters into and is common to many species of more composite bodies; for example, carbon is an essential element in all living bodies. As such, the chemical elements are species-neutral principles of nature and natural science. More fundamentally, however, the quantum physics that accounts for the specific stability (ground state, allowed transitions, chemical bonds, and reactivity) of atoms is species-specific and thus holistic, unlike classical physics.[80]

80. A great question is posed: as we ascend from the more elementary and potential to the more composite and actual levels of nature (from nuclei and electrons to atoms, to molecules, to gases, liquids, solids, cells, tissues, organs, organisms) where does the holism “top out,” i.e., at what level do we have a whole complete in itself and independent of some larger whole; at what level do we have a *substance*? Aristotle’s answer is commonsensical: “a human being or a plant or something of that sort … we most of all call substances.” *Metaphysics* VII.7.1032a20 (128). But there is much more to the story as, e.g., Plato, Spinoza, Hegel make clear, not to mention Aristotle’s own account of the substance that is most fully actual.

In sum: in view of the twofold problem of atomic stability, it is not surprising that the new type of theory required to account for the phenomena does not possess the three fundamental characteristics of all classical physics: (1) continuity of space, time, and motion, (2) spatio-temporal imageability of elementary processes, and (3) deterministic causality. The next step—logical and chronological—in the story would be quantum physics and its non-classical characteristics.[81] This vast topic is very well covered in many works, and lies outside the natural-philosophic and phenomenological intention of the present essay.[82] Accordingly, I return to the mathematization of nature, which obviously extends to both classical and quantum physics.

PART 8—PHYSICO-MATHEMATICAL SECULARISM: WORKING AROUND THE QUESTION OF THE DIFFERENCE BETWEEN MATHEMATICAL OBJECTS AND PHYSICAL OBJECTS

Are mathematical objects different in some fundamental way from physical objects? Plato says, yes: as intelligible, mathematical objects must exist independently of all sensible (material and changeable) things. Aristotle says, yes, but not in the way that Plato thinks: mathematical objects do not exist independently of sensible things but they can be understood independently of them through abstraction.[83] Descartes says, no, the object of physics, matter in motion, *is* the object of geometry, figurate extension.[84] My point is simply that there are well-known major disagree-

81. The continuous but non-spatio-temporal time-evolution of the wave function punctuated by its discontinuous "collapse" in the act of measurement; superposition and interference, Heisenberg indeterminacy and Bohr complementarity, non-locality or entanglement. See note 10 (above).

82. For an extensive bibliography (covering all interpretations, not just Copenhagen) see Bryce S. Dewitt and R. Neill Graham, "Resource Letter IQM–1 on the Interpretation of Quantum Mechanics," *American Journal of Physics* 39 (July 1971): 724–36; and John A. Wheeler and Woyciech H. Zurek, eds., *Quantum Theory and Measurement* (Princeton, N.J.: Princeton University Press, 1993). For a good account written for laymen, see David Lindley, *Where Does the Weirdness Go?* (New York: Basic, 1996), esp. 129–68. For a useful article-length overview, see Max Tegmark and John A. Wheeler, "100 Years of Quantum Mysteries," *Scientific American* (February 2001): 68–75.

83. For example, Plato, *Republic* 510c–e, 525de, 529b; Aristotle, *Physics* 193b23–194a2, and *De Anima* 431b13–18.

84. See note 25 (above).

ments in the history of philosophy about the being of mathematical objects and their relation to physical objects. In the face of these long-standing disagreements, Newton represents a new position: let us set aside these philosophical disputes, and *assume that any difference between mathematical objects and physical objects makes no difference for the conduct of our mathematical physics.* Henceforth, one can have one's private beliefs about the modes of being of mathematical and physical objects, such as central forces, but no scientific attention will be paid to the question. It will suffice to focus on the mathematical principles of natural philosophy; other principles (and causes) need not be discussed. Newton does not explicitly assert the italicized words, above, but his posture is fairly clear from the content (not to mention the title) of the *Principia*, beginning with the Preface, and it is especially well brought out by de Gandt who introduces the term "secularism" to characterize it.[85] (One sees the analogy to secularism in early modern political theory: one's religious beliefs will be a private matter; the differences between, say, Catholics and Protestants will not be allowed to affect the conduct of government.) In the following, I explicate the meaning of physico-mathematical secularism in terms of (1) Cartesian coordinates in the classical framework of space and time, (2) the Heisenberg uncertainty principle in quantum physics, and (3) act and potency in the physics of Aristotle.

Physico-mathematical secularism is embedded in the use of Cartesian coordinates (x, y, z, t) to represent *physical* space and time, and physical properties of bodies, particles, and fields, as described in the opening lines of de Broglie's description. Consider the particle trajectory that we then calculated and represented on paper in part 2: at each instant of time, t, the particle is conceived to possess a real-numerically precise value of position, x, y, z, and a real-numerically precise value of

85. *Force and Geometry*; see note 7 (above). In Def. VIII, concerning central forces, Newton says, "I here design only to give a mathematical notion of those forces, without considering their physical causes and seats" (*Principia*, 5). The preface of the *Principia* is more subtle (and remarkable): the first third explains that both mechanics and geometry should be understood in terms of (and subsumed under) *accuracy*. Accuracy means the fit or match (see note 16, above) between (1) a perfect figure ("perfectly accurate") and one drawn less perfectly, as well as (2) the closeness of a calculated to a measured number of units ("accurately proposes and demonstrates the art of measuring"), i.e., numerical precision in terms of decimal digits, e.g., accurate to the fifth decimal place. Thus when Newton (remarkably) says, "the errors are not in the art, but in the artificers," I take him to mean that there is no mismatch between the mathematical and the physical *that is rooted in the nature of the physical itself.*

momentum, mv_x, mv_y, mv_z, relative to the center of force. In general, it is assumed that such variable magnitudes can faithfully represent anything measurable in the physical world. It is thus presupposed that the properties of physical objects are conformable to the real-number line, or that the real-number line is perfectly adequate to the properties of physical objects.[86] Our measurements (using instruments of increasing precision) will match (with increasing accuracy) our calculations (solving the equations that express the laws of nature). We can then predict, and, to the extent possible, control the physical quantities that we conceive as objectively existing in space and time, e.g., particle position and momentum. Or are we confusing mathematical objects with physical objects? No matter (pun intended); it is not a problem: unwitting reification of mathematical objects can do no harm (lead to no fundamental error) in physics—this follows from the original (seventeenth-century) assumption of physico-mathematical secularism.[87]

This assumption is dubious, the more so in view of the highly constructed character of the Cartesian, numerical-variable magnitudes that

86. The real number system was brought to explicit definition in the nineteenth century through the work of Dedekind and Cantor, and so Descartes and Newton did not use the terms "real number" or "arithmetical continuum." The absence in their work of any concern for the longstanding premodern doctrine of the essential heterogeneity of discrete number and continuous magnitude makes clear that their conception of number and magnitude was implicitly real-numerical, e.g., the variable, x, is both a number and a line segment, thus the expression, $1 + x + x^2 + x^3 + x^4 + \ldots$ ($x<1$), as in Newton's analyses of infinite series, makes sense not only arithmetically but also geometrically as it could not in classical geometry.

87. The significance of non-linear dynamics or "chaos theory" is that, while leaving the classical concept of deterministic trajectory intact, it corrects the assumption of the unconditional conformability of measurable properties of physical objects to the mathematical real number line. *As measurable*, physical properties can be numerically specified only approximately, e.g., to within Δx, finite no matter how small. Therefore, if a non-linear physical system is in the regime of sensitive dependence on initial conditions, then, due to the rapid divergence of initially adjacent (within Δx) trajectories, no actual measurement, e.g., of initial position, $x(t_0) \pm \Delta x$, can determine a unique trajectory. "Newtonian dynamics has, over the centuries, twice foundered on the assumption that something was infinite when in fact it was not: the speed of light, c, and the reciprocal of Planck's constant, 1/h. Reformulations omitting these infinities led first to special relativity and then to quantum mechanics. Complexity theory now reveals a third tacitly assumed infinity in classical dynamics, namely the assumption of infinite computational and observational precision." Joseph Ford, "How Random Is a Coin Toss?," *Physics Today* (April 1983): 40–47, here 46. Another point, more centrally relevant to my purpose: If physico-mathematical secularism arose in the seventeenth century, what about the preceding Aristotelian-scholastic physics and mathematics, which spoke of mixed sciences; what is the relation between the mixed-science tradition and physico-mathematical secularism? This important question is discussed in the conclusion.

FIG. 3

we use in our physics. To see what I mean by "highly constructed character," look at the two magnitudes (lines) shown in boldface:
Are the two equal or unequal? The answer is, yes. As Euclidean geometrical magnitudes, they are obviously unequal, since I can lay the left one adjacent to the right one, cut off a part of the right one equal to the left one, and exhibit the remainder.[88] But as Cartesian variable magnitudes, taken in our minds as possessing (unlike Euclidean magnitudes) *numerical* value, point by point, based on our arbitrary choice of a unit length, they are obviously equal, since they both have the same numerical value, c. We, unlike premodern mathematicians, have two different concepts of magnitude in our mental toolbox. The difference between them is considerable.

The continuous magnitudes of Euclidean geometry are "put before us" in a direct way.[89] A line, for example, is delimited by its two endpoints (each designated by a letter) within a given figure, and it is the definition or *logos* of the figure from which the intelligibility or meaning, thus the *being* of the line—how it is related (by equality, proportionality, parallelism, perpendicularity, etc.) to the other parts of the figure—*begins*. The line has no numerical character; it is not a *length*, that is, a number of units of measurement. Nor are two points separated by a *distance* (the length of the line they delimit). Accordingly, Euclidean demonstrations do not involve the operations (+, −, ×, ÷, √) to which numbers are subject.[90] Finally, we must continually look with our eyes at the drawn

88. *Elements*, common notions 4 and 5.

89. I owe to Andrew Romiti the very helpful phrase "put before us."

90. More precisely: Euclidean magnitudes of the same kind can be added unconditionally. They can be subtracted on condition that the one removed does not exceed that from which it is taken. "Multiplication" (called "application") is limited to construction of a rectangle from two lines or a solid from a line and a plane figure. Division of heterogeneous magnitudes (a

figure in front of us and examine it, but we know that our thinking is not about that figure but about an ideal, intelligible one, of which the visible one is an imperfect image.

The numerical-variable magnitudes or *coordinates* of Cartesian analytic geometry are quite different. Here, everything begins (consider plane problems) from the x and y *axes*. A point on an axis, say, the x-axis, is conceived as having, not a particular numerical value, such as 53.74, but numerical value in general, x, which stands for the distance from the origin that it would have if a unit of length were chosen on the axis. The origin does have a particular value: 0. Since a unit length can always be chosen (place a mark on the axis to the right of 0) whereby any line acquires a numerical value in terms of that unit (ratios of incommensurable lines having been conceived as irrational numbers), we need not choose a unit prior to performing calculational operations, +, −, ×, ÷, √, on the letter signs, x, y, a, b, etc. A unit can be specified later, if needed. We are working with generic numbers (increasing to the right on the x-axis or upward on the y-axis from the origin). The x-y plane is then brought into being by our mental conception of its points as ordered pairs, that is, as coordinates (x, y), where x means the distance the point would have from the y-axis measured parallel to the x-axis, and y means the distance it would have from the x-axis measured parallel to the y-axis upon choice of a unit. Most relevant and useful: the coordinates, x and y, are conceived *to vary* (lengthen and shorten) relative to the origin at (0, 0). We can stipulate and express how, say, y varies with x in a *functional relation*, such as y(x) = ax + b. This *equation* is usefully expressed by the curve (here a line) whose points possess the coordinates (x, y(x)) thereby defined. Curves and their equations can be studied, and problems can be solved by operating on the letter-signs and finding the algebraic expressions for the functional relation between coordinates of interest, for example, the position and momentum of a particle in relation to time, x(t), p(t). The Cartesian letter signs, or symbols, x, y, t, a, b, etc., for variables and constants do not image ideal objects as do the figures of Euclidean geometry. Nor do they stand for the definite numbers of pure units (monads) or their fractional parts as in the Diophantine arithmetic.

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rectangle by a line) is impossible. Ratio can exist only between two magnitudes of the same kind; accordingly, a proportion involving heterogeneous magnitudes cannot be alternated.

My point in all of this is that the mind's constructive activity contributes much to the structure of Cartesian geometry—to the intelligibility or meaning of its thus conceptually complex objects. (Note the appearance of the words "conceived" and "conception" in the preceding paragraph; they have the sense defined in Descartes's *Rules for the Direction of the Mind*.)[91] The Cartesian plane, and the coordinate axes that bring it into being are not "put before us" in the direct manner of Euclidean magnitudes.[92] Now can we *identify* the spatio-temporal objects of physics with these conceptually constructed objects of mathematics? Is the conformability of the former to the latter, or the adequacy of the latter to the former unconditional—as was assumed by the physico-mathematical secularism of classical physics? My repeated reference, above, to the position and velocity or momentum, thus the trajectory of a particle should make the answer to this question quite clear. The Heisenberg uncertainty principle is emblematic of the failure of physico-mathematical secularism.

In 1927 Werner Heisenberg set out what has since been known as the uncertainty principle of quantum physics: "The more accurately the position [of a particle] is determined, the less accurately the momentum is known and conversely."[93] More specifically: the product of the uncertainty in the measurement of particle position, x, at time, t, and the uncertainty in the measurement of its momentum, Δp, at time, t, cannot be reduced below (approximately) the numerical value of $\hbar$, Planck's constant divided by 2π: $\Delta x \Delta p \geq \hbar$. The reciprocal relation of the uncertainties means that, as position, x(t), is measured with increasing accuracy—approaching the classical ideal, $\Delta x = 0$—all values of the momentum, p(t), become equally possible, and conversely, as $\Delta p \to 0$, $\Delta x \to \infty$. The Δx and Δp are not observational errors of classical type resulting from the imprecision of present-day instruments, to be progressively sharpened in the future. Most importantly, the Δx and Δp are not merely expressions

91. Rule 12 (AT X 416, CSM I 42). The concept of general magnitude—the progenitor of real-numerical variable magnitude or coordinate axis—is then elaborated in Rules 13–16.

92. Using the scholastic terminology of first and second intentions, Klein characterizes Cartesian magnitudes as second intentions taken by the intellect as first intentions by means of the visible letter sign, which thereby becomes an algebraic *symbol*, and analytic geometry an algebra of line segments. See *Greek Mathematical Thought*, 208, and *Lectures and Essays*, 17–21.

93. "Je genauer der Ort bestimmt ist, desto ungenauer ist der Impuls bekannt und umgekehrt." Heisenberg, "Über den anschaulichen Inhalt der quantentheoretischen Kinematic und Mechanik," *Zeitschrift für Physik* 43 (1927): 127–98, here 175.

of an unpredictable disturbance of the tiny particle's, such as the electron's, local motion due to the unavoidably much larger (and energetic) observational apparatus. This is the "uncontrollable disturbance" interpretation of the uncertainty principle. It leaves the central concept of classical mechanics—particle trajectory, that is, simultaneously well defined values of position and momentum—intact, maintaining only that the trajectory is made unpredictable in the future by our unavoidably intrusive efforts to observe the particle right now (it gets uncontrollably deflected by the light used to detect it), so that the trajectory is uncertain only to us, not indeterminate in itself. Rather, the uncertainty principle is ontological, such that, "the term 'uncertainty principle' is, therefore, somewhat of a misnomer. A better term would be 'the principle of limited determinism in the structure of matter,'" or the Heisenberg indeterminacy principle.[94]

Heisenberg's principle means, among other things, that the being and knowability of spatio-temporal properties, like particle position, x, at time, t, are intertwined with the being and knowability of dynamical properties, like momentum, p, and energy, E. Here, again, is de Broglie:

94. David Bohm, *Quantum Theory* (New York: Dover, 1979), 101. The following is from Max Jammer, *The Conceptual Development of Quantum Mechanics* (New York: McGraw-Hill, 1966), 329–30: "[A] notion like 'path' or 'trajectory' ('Bahn'), which presupposes a sharp knowledge of both position and momentum, can be retained in quantum mechanics, contended Heisenberg. For 'the path comes into existence only when we observe it,' he declared. This contention, one of the most provocative statements ever made in physics, was justified by him as follows. If an atom which, say, is in its 1000th excited state is illuminated by light of relatively long waves [. . .] the electron, owing to the Compton effect [photon-electron scattering], may be found after the interaction somewhere between the 950th and the 1050th state, the particular orbit being indeterminate. The electron will therefore be represented by a wave packet in configuration space, composed of the eigenfunctions of the states just mentioned, and of a size determined by the precision of the position measurement, that is, by the wavelength of the illuminating light. The packet describes an orbit analogous to that of a classical particle, but spreads with time. Every new observation by which a certain [position, x] is selected out of a multitude of possibilities reduces the otherwise spreading wave packet again to the dimensions of the wavelength of the incident light. The constant replacement in each consecutive observation of the wave packet, which has spread out, by a smaller packet gives rise to the orbit as the temporal sequence of the locations where the wave packet has been observed." For the quantum mechanical explanation of a track in a cloud chamber, see Bohm, *Quantum Theory*, 137–40. Heisenberg indeterminacy and Bohr complementarity are the essential features of the Copenhagen Interpretation of quantum mechanics. Alternative interpretations that accord with the classical conception of mind and world are, after Bell's theorem and the Aspect experiments, necessarily non-local and, in the case of Bohm theory, holistic with a vengeance; see D. Bohm and B. J. Hiley, *The Undivided Universe* (London: Routledge, 1993).

What is now [e]specially important for us to understand is the profound meaning of this rather mysterious idea of the quantum of action [Planck's constant, $h = 6.62 \times 10^{-27}$ erg-sec]. Up till [the early twentieth century] the space and time of classical physics, or its successor—the space-time of the relativity physics—had appeared to us as a framework given *a priori* and [being] quite independent of what one could put into it, [being] quite independent particularly of the movements and evolution of the bodies which were localized in it. [...]

The real significance of the quantum of action has been disclosed to us notably by the discovery of Heisenberg's uncertainties [...]. It seems certain today that the existence of the quantum of action expresses a formerly totally unsuspected union between the framework of space and time and the dynamical phenomena which take place in it. The picture of space and time [in classical physics] is essentially static; a body, a physical entity, which has an exact location in space and in time is, by this very fact, deprived of all evolutionary property; [but] on the contrary, a body which is developing, which is endowed with dynamic properties, cannot really be attached to any point of space and time. These are philosophical remarks which go back to Zeno [and so to Aristotle, *Physics* VIII.8]. [...] Heisenberg's uncertainty relations appear akin to these remarks; they teach us, in effect, that it is impossible to attribute simultaneously to a body a well-defined motion and a well-determined place in space and time.[95]

There is no way in the classical conception of mind and world, space and time, mathematics and physics, measurement and calculation, in which our knowledge of one physical quantity, say, position or time, could affect or interfere with, or limit our knowledge of another such quantity, say, momentum or energy. They are all just Cartesian magnitudes, real numbers of appropriate units. They can be thought conjointly and put together on paper (the position and velocity vectors in the trajectory calculation); or separated in our thought and on paper. As Cartesian mathematical objects, each is unmodified by, and indifferent to its membership in any whole. There is no "holism" of motion, mobile, place, and time. But, as de Broglie reminds us, pointing back to Aristotle, a moving body—as opposed to a mathematical point "moving" in our imagination—is not *actually* in a place or at a fixed position; if it were, it would not be in motion. Matt Crawford puts it nicely:

95. De Broglie, *Physics and Microphysics*, 120–22

The identification of natural beings, having first intentional magnitudes (as Klein says), with Cartesian real-numerical magnitudes (by which identification they are also reduced to a collection of decoupled, quantity-holding attributes, abstracting from other [. . .] characteristics that depend on the coupled whole), is no longer tenable given that, as we now know, the framework of space and time is only putatively determinate in the way real numbers are, and this putative determinateness is an artifact of its abstract conception.[96]

Aristotle, of course, did not quantify the indeterminacy in the position of a body moving with a given speed; he did not discover Planck's constant. But Aristotle does prepare us for the ideas of indeterminacy and potentiality—of some things having less being than others—and thus of limits to the intelligibility of the potentially being. Heisenberg used Aristotle as an aid in attempting to explain the obscure reality of the wave function: it is "a quantitative formulation of the concept of δύναμις, possibility, or in the later Latin version, *potentia*, in Aristotle's philosophy."[97] The actualization of this potentiality is the act of measurement whereby the wave function is reduced, or "collapses" to one of the eigenstates of which it is a superposition (a type of sum) and to which corresponds a possible (potential) result of the measurement, for example, the particle has spin up or spin down. Heisenberg's analogy to Aristotle is right in this sense: the process of wave-function collapse cannot be expressed mathematically, it cannot be modeled (imaged) as the spatio-temporal variation of some Cartesian magnitudes as in a classical equation of motion. Wave function collapse from a set of many possible outcomes to one actual outcome escapes the grasp of mathematics. With a notable exception, discussed below, the reduction of *dunamis* to *energeia* in Aristotle's physics likewise escapes the grasp of mathematics. Unlike Aristotle's act and potency, however, the possible outcomes

96. Matt Crawford, private communication, July 30, 2006.

97. The full quotation is that the probability amplitude, $\psi(x, t)$, is "a quantitative formulation of the concept of δύναμις, possibility, or in the later Latin version, *potentia*, in Aristotle's philosophy. The concept that events are not determined in a peremptory manner, but that the possibility or 'tendency' for an event to take place has a kind of reality—a certain intermediate layer of reality, halfway between the massive reality of matter and the intellectual reality of the idea or the image—this concept plays a decisive role in Aristotle's philosophy. In modern quantum theory this concept takes on a new form; it is formulated quantitatively as probability and subjected to mathematically expressible laws of nature [e.g., the Schroedinger equation]." Heisenberg, *On Modern Physics* (New York: Clarkson Potter, 1961), 9–10.

of a quantum measurement occur with numerical probabilities that can be calculated from the wave function, such as .75 for spin up, .25 for spin down. More generally (since every analogy is imperfect), of the various senses of the potentially being and its actualization in Aristotle, none fits quantum processes exactly.[98]

Let us state the essential conclusion of this section: because act and potency, thus actualization of potentiality and material parts potential in the whole, do not belong to mathematical objects—whether ancient or modern—modern physico-mathematical secularism entails their banishment from physics. This decision was adequate for classical physics, but not for quantum physics.

CONCLUSION: SEEING HETEROGENEITY, MAKING HOMOGENEITY

The preceding paragraph poses a final question on which to collect our thoughts and finish this essay: premodern (ancient and medieval) physics had a significant mathematical component, namely, the four mixed or intermediate sciences, optics, harmonics, mechanics, and astronomy, which employed premodern mathematics, namely, arithmetic, geometry (also trigonometry), and proportion theory.[99] For example, Aquinas speaks of "reckoning the courses of the stars" (*cursus siderum computare*) in mathematical astronomy, for "mobile and incorruptible beings, owing to their uniformity and regularity, can be determined in their movements by mathematical principles."[100] But there is nothing in premodern

98. I find five senses (some overlapping) of potentiality with respect to material structure and change in Aristotle: (1) motion (*kinêsis*) in the categories of quantity, quality, and place, and (2) change (*metabolê*) in the category of substance (*Physics* III.1.201a11, 210a29, 201b5, III.2.202a7; *Metaphysics* V.12.1019a21–23, VII.9.1034a35–b1, IX.7.1049a14–17); actualization of (3) first and (4) second potency (*Physics* VIII.4.255a32–b14, *De Anima* II.1.412a22–29); (5) body parts being potentially in a whole living substance (*Metaphysics* V.26.1023b33–35, VII.16.1040b5–17; *De Anima* II.2.414a20–28). With respect to mathematical objects, points and intervals can be potentially in a line (*Physics* VIII.8.262a22–24, 263a28). For a detailed and clear explanation of the sense of potentiality in quantum physics, see Bohm, *Quantum Physics*, 132–33, 138–40, 166–67. This is not to be confused with the quantum potential of Bohm's later hidden-variables (specifically pilot-wave) theory; "A Suggested Interpretation of the Quantum Theory in Terms of 'Hidden' Variables," *Physical Review* 85, no. 2 (January 15, 1952): 166–93.

99. Aristotle, *Posterior Analytics* I.13.78b35–79a16; *Physics* II.2.193b32–194a12.

100. Aquinas, *In de trinitate*, q. 5, a. 1, ad 3, and a. 3, ad 8 (18, 46). In "reckoning the courses of the stars," geometrical figures (Ptolemaic circles) would image their paths while numbers

science like physico-mathematical secularism, no suspension of judgment concerning the difference between mathematical objects and physical objects, thus no ruling (methodological) assumption that the difference between them could make no difference for the science of nature. What accounts for this? The answer to this question becomes clear to the extent that we can remove ourselves from the conceptual standpoint of modern mathematics and physics and recover that naively direct (and thus error-prone) way of receiving the world characteristic of Aristotle's philosophy. There, as noted in part 3, above, we see (literally and figuratively) *heterogeneity*—differences in kind—in both mathematical beings and physical beings. As long as we apprehend several different kinds of mathematical beings (continuous magnitudes, discrete numbers, and, where homogeneity restrictions permit, ratios and proportions of them) none of which is subject to *kinêsis*, along with several other different kinds of physical beings (celestial, terrestrial, and among the latter, nonliving and living, and among the latter, plants and animals, and among the latter, nonhuman and human) each of which is substance and subject to *kinêsis* in different analogous degrees—as long as this is the case, it is clearly impossible to think that the difference between mathematicals and physicals might not matter for the science of nature.

The most striking difference or heterogeneity in the physical world is that between celestial and terrestrial. Aquinas just mentioned the most perfect kind of physical beings, the celestial bodies, which are (by the erroneous science of his time) incorruptible, and whose local motions are, accordingly, so regular that arithmetic and geometry can be applied to them. But, therefore, among the celestial bodies, generation does not occur, but only local motion, in which the mobile "departs least from its substance" (*Physics* VIII.7.261a22). In striking contrast, among the terrestrial living things, there is a craftlike succession of stages in their generation from seed (*Physics* II.8.199a8–19); final cause and chance are evident there, but there is (to this day) no mathematical description of their natural local motions. In sum, celestial and terrestrial beings are vividly distinct: in the one kind, we have mathematical description but final cause is not evident; in the other, final cause is evident but we have no

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and fractions of angular units (all discrete) would approximate the positions of the stars at a given time continuously moving on those paths.

mathematical description. Among mathematical beings, as opposed to physical, the fundamental difference in kind is not at all so evident.

The essential heterogeneity among mathematical beings is that of discrete number and continuous magnitude. It is very difficult for us today to understand sympathetically the reasons for this longstanding premodern distinction. We are, as it were, born to the real number line, the Cartesian numerical-variable magnitudes described above, and this mental formation begins with elementary-school arithmetic. Jacob Klein's work on the origins of algebra is about this, and thus about the removal of the distinction within human cognition between discrete and continuous mathematical beings in favor of algebraic *symbols* lying in "a new [homogeneous] conceptual dimension"—and all that this implies.[101]

I wish here simply to bring out the following basic point: it is only after homogenizing the heterogeneities of both physical and mathematical objects, and thus conceiving the possibility of an adequation, fit, or match of Cartesian numerical magnitudes to the properties of physical beings, that physico-mathematical secularism becomes possible. These two homogenizations are historically concurrent but very different. The one in physics or natural science is famous; it is the Scientific Revolution, culminating in Newton's physics and the accompanying forces-and-particles conception of the world in which species-specific forms and ends are replaced by species-neutral laws of nature. The one in mathematics remains all but invisible.[102] But both have this in common: they have more the character of making (construction, creation) than of seeing (intuition, intellection).[103] But whereas the homogenizing transition in mathematics is subtle and inconspicuous, that in physics seems conspicuously willful. Granted, it was surely right to remove the mis-

101. "[This] modification … of ancient mathematics is *exemplary* for the total design of human knowledge in later times" (*Greek Mathematical Thought*, 120–21). "Therewith the most important tool of mathematical natural science, the 'formula,' first becomes possible … but, above all, a new way of 'understanding,' [a different conception of the world, a different understanding of the world's being] inaccessible to ancient *episteme* is thus opened up" (ibid., 175, 152). See 184–85, 192, 213, on the awareness of fundamental problems that is thereby lost.

102. "It remains immensely difficult to leave that medium of ordinary intentionality which corresponds to our [modern] mode of thinking, a mode essentially established in the last four centuries" (*Greek Mathematical Thought*, 118).

103. For a remarkable and remarkably interesting development of the meaning of construction in the history of mathematics, see David R. Lachterman, *The Ethics of Geometry: A Genealogy of Modernity* (New York: Routledge, 1989).

taken heterogeneity of celestial and terrestrial, but was it right to remove *all* heterogeneity from nature (living and nonliving, *human and nonhuman*), thereby committing science to conceive nature as all one stuff, for example, forces and particles, or particles and fields—"material to work on" in Locke's memorable phrase?[104] As long noted by thoughtful commentators, this leaves human being, especially the scientist, in an odd position.[105] Perhaps the greatest value of Aristotle lies not his philosophy of nature, which is valuable, as I have tried to show, but in his ethics: "Precision ought not to be sought in the same way in all discourses.... For it belongs to an educated person [*pepaideumenou*] to seek just so much precision in each kind [of discourse] as the nature of the subject admits."[106] But Aristotle's educated person has a special disposition that enables perception and judgment of limits and boundaries, one that is difficult to reconcile with that of modern natural science. And so we return to the beginning of this essay. I give Heidegger almost the last word:

Modern science's way of representing pursues and entraps nature as a calculable coherence of forces. Modern physics is not experimental physics because it applies apparatus to the questioning of nature. Rather the reverse is true. Because physics, indeed already as pure theory, sets nature up to exhibit itself as a coherence of forces calculable in advance, it therefore orders its experiments precisely for the purpose of asking whether and how nature reports itself when set up in this way.... If modern [quantum] physics must resign itself ever increasingly to the fact that its realm of representation remains inscrutable and incapable of being visualized, this resignation is not dictated by any committee of researchers. It is challenged forth by the rule of Enframing [*Ge-stell*], which demands that nature be orderable as standing-reserve. Hence physics, in all its retreating from the representation turned only towards objects [i.e., the classical conception of

104. Locke, *Second Treatise of Civil Government*, chap. 5, section 35.

105. In addition to the writings of Kennington and Riezler, cited above, see the reflections on natural science and human self-understanding in Hans Jonas, Leon Kass, Charles De Koninck, C. S. Lewis, and Leo Strauss.

106. Nicomachean Ethics I.3.1094b13, 24–25 (2, translation slightly modified). Rule 1 of Descartes's *Rules for the Direction of the Mind* is precisely targeted against this (AT X 359–60, CSM I 9). The remainder of that early, unfinished work is a key text for the twofold homogenization that is constitutive of the modern scientific mind. We can appreciate Heidegger's remark: "Only one who has really thought through this relentlessly sober volume long enough, down to its remotest and coldest corner, fulfils the prerequisite for getting an inkling of what is going on in modern science." *What Is a Thing?*, trans. W. B. Barton Jr. and Vera Deutsch (South Bend, Ind.: Regnery, 1967), 101.

mind and world] that has alone been standard till recently, will never be able to renounce this one thing: that nature reports itself in some way or other that is identifiable through calculation and that it remains orderable as a system of information [i.e., digit strings].[107]

This means that our physics has nothing to say about the unity of nature or creation.

107. Heidegger, *The Question concerning Technology*, trans. William Lovitt (New York: Harper and Row, 1977), 21, 23.