History of Physics and the Thought of Jacob Klein

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Abstract: Aristotelian, classical, and quantum physics are compared and contrasted in light of Jacob Klein's account of the algebraicization of thought and the resulting detachment of mind from world, even as human problem-solving power is greatly increased. Two fundamental features of classical physics are brought out: species-neutrality, which concerns the relation between the intelligible and the sensible, and physico-mathematical secularism, which concerns the question of the difference between mathematical objects and physical objects, and whether any differences matter. In contrast to Aristotelian physics, which is species-specific, classical physics is species-neutral. In contrast to both Aristotelian and quantum physics, classical physics assumes that any differences between mathematical objects and physical objects make no difference for the conduct of physics. Aristotle's act and potency, and Heisenberg's uncertainty principle are discussed as counterexamples to the physico-mathematical secularism of classical physics. The algebraicization of thought in conjunction with the disposition and program for the mastery of nature leads to the homogenization of heterogeneities in both mathematics and physics, and, therewith, to confusion concerning the meaning of human being and our place in the whole.

Keywords: Aristotelian physics; classical physics; intentionality; Jacob Klein; quantum physics; symbolic thinking.

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> The creation of a formal mathematical language was of decisive significance for the constitution of modern mathematical physics. … [T]his study … will confine itself to the limited task of recovering to some degree the sources, today almost completely hidden from view, of our modern symbolic *mathematics*. Nevertheless, the inquiry will never lose sight of the fundamental question, directly related as it is to the conceptual difficulties arising within mathematical physics today. However far afield it may run, its formulation will throughout be determined by this as its ultimate theme.[2]

Introduction

With his unique work on the history of mathematics much in mind, I offer some reflections on the physics that provides Jacob Klein's remote but ultimate theme. The principal device employed in the following is a comparison between classical physics, on the one hand, and Aristotelian physics, on the other. From the contrast between them, two fundamental features of classical physics come into view: Species-neutrality,[3] and physico-mathematical secularism. The laws and equations of classical physics are species-neutral in contrast to the Aristotelian principles of form and matter, which are species-specific. The contrast between Aristotelian and classical physics here concerns the relation between the intelligible and the sensible.

The second fundamental feature of classical physics is physico-mathematical secularism, which is about the relation between mathematics and natural science. I take the term "secularism" in this connection from François De Gandt on Newton.[4] Physico-mathematical secularism concerns the questions of how and whether mathematical objects and physical objects differ and, specifically, whether it matters. Classical physics, in contrast to Aristotle's physics, assumes that it does not matter. I explain this in the following. Both species-neutrality and physico-mathematical secularism make significant contact with the thought of Jacob Klein.

2. Jacob Klein, *Greek Mathematical Thought and the Origin of Algebra*, trans. Eva Brann (Cambridge, MA: MIT Press, 1969; reissue: New York: Dover, 1992), 18, 19. This work was originally published in German as "Die griechische Logistik und die Entstehung der Algebra" in *Quellen und* Studien *zur Geschichte der Mathematik, Astronomie und Physik*, Abteilung B: *Studien*, vol. 3, no. 1 (Berlin, 1934), 18–105 (Part I); no. 2 (1936), 122–235 (Part II), 3, 4. Henceforth the English translation is cited as *GMTOA* followed by page number.

3. The general idea of laws of nature as species-neutral principles is pioneered by Bacon. See *New Organon*, I.51, I.66, I.88, II.3, II.35, ed. Fulton H. Anderson (Indianapolis, IN: Bobbs-Merrill, 1960), 53, 63, 86, 122, 186, and Richard Kennington, *On Modern Origins*, ed. Pamela Kraus and Frank Hunt (Lanham, MD: Lexington Books, 2004), 25, 53–4, 257.

4. "Newton claimed to treat forces in a purely mathematical mode; by deferral, which in a sense turned out to be final, he left in suspense the properly philosophical or physical questions concerning the causes of gravitation and the ontological reality of force. This neutrality (or 'secularism') of centripetal force in face of the controversies on the cause of gravitation is the essential characteristic of the new science." De Gandt, *Force and Geometry in Newton's* Principia, trans. Curtis Wilson (Princeton, NJ: Princeton University Press, 1995), x–xi.

Classical physics and its associated conception of the material world are famously exemplified by Newton's theory of the solar system and its generalization to the forces-and-particles model of the universe, a vast mental construct at the heart of which is the concept of *trajectory*—the path in space of a body moving through time under laws of motion and force, starting from a given position and velocity.[5] Certain basic and very general characteristics of this Newtonian account are found throughout all of prequantum physics, including thermodynamics, the field physics of Maxwell, the relativity physics of Einstein, and the recently developed non-linear dynamics or "chaos theory" originally discovered by Poincaré.[6] The best description known to me of classical physics in this sense—that is, all post-seventeenth-century mathematical and experimental physics, with the

5. The starting position and velocity of the body are "given" in two senses: first, as stipulated or set down by the theoretical physicists in order to initiate the calculation of the trajectory under the equations of motion (to see how it behaves); second, as measured by the experimental physicists using instruments (like meter sticks, scales and clocks). The concept of trajectory echoes the following words of Klein: "The new science now understands just this lawfulness in the *course* of motion, in the *temporal* sequence of states of motion, as the order of the world. The order of things moves up one story higher, so to speak, when the temporal dimension is added. … Above all, the concept of conformity to law signifies a modification of the ancient concept of τάξις [good order of the cosmos that is time-independent]; τάξις is now understood as *lex*, that is, as order over time. The ascent from *prima intentio* to *secunda intentio* is initiated here by the insertion of the time dimension [in Galileo's law of fall, $s_1 : s_2 = t_1^2 : t_2^2$, and Kepler's third law, $t_1^2 : t_2^2 = r_1^3 : r_2^3$]." "The World of Physics and the 'Natural' World," in *Jacob Klein Lectures and Essays*, ed. Robert B. Williamson and Elliott Zuckerman (Annapolis, MD: St John's College Press, 1985), 1–34, here 33 and 34. But is the transition from τάξις to *lex* a matter of symbolic concept formation or the creation of a formal mathematical language?

6. Henri Poincaré, *Science and Method*, trans. Francis Maitland (New York: Dover, 1952), 67–9. Chaotic dynamics falls within classical physics because the mathematical models (non-linear differential or difference equations) hypothesized to account for certain unpredictable physical phenomena (e.g., a forced pendulum, turbulent flow, weather patterns) yield deterministic trajectories, as discussed in Part 1, below. What distinguishes chaotic dynamics within classical physics is the following secondary but significant point concerning the initial data, i.e., the numerical values inputted to the model (the first sense of "given" in note 5, preceding) that specify a particular deterministic trajectory: Due to sensitive dependence on initial conditions—a feature of non-linear equations—no matter how close the numerical values with which two modeled trajectories begin, they (the two trajectories) diverge so rapidly that even the most precise *empirical* measurement of initial data, e.g., on turbulent flow, a weather system, a forced pendulum (the second sense of "given" in note 5) would still contain within its tiny but necessarily finite error interval differences in numerical value that would determine trajectories so rapidly diverging that the physical process would *appear* random, i.e., under the same empirically measured conditions, very different physical changes occur no matter how precise the measurements. Thus, any systems in nature governed by nonlinear interactions of this type (subject to sensitive dependence on initial conditions) would be fully deterministic but quite unpredictable in their behavior. Hence the name "deterministic chaos" is often used to describe the field of non-linear dynamics; see Heinz Georg Schuster, *Deterministic Chaos* (Weinheim, Germany: VCH Verlagsgesellschaft, 1989), 1–5. For a good nontechnical presentation, see David Ruelle, *Chance and Chaos* (Princeton, NJ: Princeton University Press, 1991), also James Gleick, *Chaos: Making a New Science* (New York: Viking Penguin, 1987). See note 61, below, on the philosophical significance of chaos theory.

exception of quantum theory—is given by Louis de Broglie in his 1955 *Physics and Microphysics*, from which I quote momentarily. De Broglie was an important founder of quantum physics. Against the background of classical physics, quantum physics appears indeed to be a radical departure. But classical physics is itself a radical departure from the preceding, Aristotelian understanding of nature.[7] This is hardly to say that Aristotelian natural philosophy can provide the adequate philosophical comprehension of quantum physics. The quantum phenomenon of non-locality[8] is a challenge for any philosophy of nature or science today, and, historically, the Aristotelian doctrine of the essential heterogeneity of terrestrial and celestial matter and motion was an error that impeded the progress

7. "The design of reality in classical physics contains greater enigmas than your so-called quantum mystery." Kurt Riezler, *Physics and Reality: Lectures of Aristotle on Modern Physics at an International Congress of Science* (New Haven, CT: Yale University Press, 1940), 25.

8. Non-locality or entanglement refers to a remarkable effect—instantaneous action at a distance to arbitrary range—predicted by quantum physics and highlighted in the famous 1935 exchange between Bohr and Einstein, Podolsky and Rosen (EPR), over the completeness of the Copenhagen Interpretation. Einstein, Podolsky and Rosen, "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?" *Physical Review* 47 (May 15, 1935), 777–80. Bohr, "Can Quantum-Mechanical Description of Physical Reality be Considered Complete?" *Physical Review* 48 (October 15, 1935), 696–702. We can get a sense of the issue against the background of Einstein's firmly classical conception of what physics is and ought to be: "I cannot seriously believe in [quantum mechanics] because the theory cannot be reconciled with the idea that physics should represent a reality in time and space, free from spooky actions at a distance." Einstein to Born, 1947, in Max Born, *The Born-Einstein Letters* (New York: Walker and Co., 1971), 158. For Einstein (as for many), instantaneous action at a distance is by nature impossible. Schrödinger clearly foresaw the prediction of "spooky actions at a distance" entailed by his equation for the wave function. Writing shortly after the EPR–Bohr exchange, he introduced the term 'entanglement': "When two systems, of which we know the states by the respective representatives, enter into temporary physical interaction due to known forces between them, and when after a time of mutual influence the systems separate again, then they can no longer be described in the same way as before, viz. by endowing each of them with a representative of its own [independent of the other]. I would not call that *one* but rather *the* characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought. By the interaction, the two representatives (or ψ-functions) have become entangled. To disentangle them we must gather further information by experiment. … After reestablishing one representative by observation, the other one can be inferred simultaneously. In what follows the whole of this procedure will be called *the disentanglement*. Its sinister importance is due to its being involved in every measuring process. … Attention has recently been called [Einstein, Podolsky and Rosen, *Phys. Rev*. 47 (1935), 777] to the obvious but very disconcerting fact that even though we restrict the disentangling measurements to *one* system, the representative obtained for the *other* system is by no means independent of the particular choice of observations which we select for that purpose and which by the way are *entirely* arbitrary. It is rather discomforting that the theory should allow a system to be steered or piloted into one or the other type of state at the experimenter's mercy in spite of his having no access to it. This paper does not aim at a solution of the paradox, it rather adds to it, if possible." Erwin Schrödinger, "Discussion of Probability Relations Between Separated Systems," *Proc. Cambridge Philosophical Society* 31.1 (January 1936): 555–6. EPR correlations were confirmed by the Aspect experiments in 1982. As of 2008, they have been detected to a range of 18 km with a speed (18km/Δt) that could be no less than 10,000 times that of light. D. Salart, A. Baas, C. Branciard, N. Gisin, H. Zbinden, "Testing Spooky Action at a Distance," *Nature* 454 (August 14, 2008): 861–4.

of science. Nevertheless, I believe that the Aristotelian background is valuable, perhaps indispensable, for the philosophical interpretation of physics, both classical and quantum. I will argue here that physico-mathematical secularism—specifically, its failure in the quantum revolution—affords an example of the relevance of the Aristotelian background.

1 Classical Physics: De Broglie's Synoptic Description

De Broglie:

> With [Cartesian] coordinates of space and time [x, y, z, t], classical mathematical physics was in a condition to represent in a precise way the succession of phenomena which our senses allow us to verify around us.
>
> From that moment a way opened quite naturally before theoretical physics and it boldly entered upon it. It was thought that all evolution of the physical world must be represented by quantities [e.g., the position and velocity of a discrete particle, or the intensity of a continuous field] localized in space and varying in the course of time. These quantities must render it possible to describe completely the state of the physical world at every instant, and the description of the whole of nature could thus be given by figures and by motions in accordance with Descartes' programme. This description would be entirely carried out with the aid of differential equations … enabling us to follow the localization and the evolution in the course of time of all the quantities defining the state of the physical world. A magnificent conception for its simplicity and confirmed by the successes which it has achieved for a long time! It sustained and orientated all the efforts of the great schools of mathematical physics of the nineteenth century.
>
> Assuredly not all scientists agreed to this description of the world by figures and movements exactly in the same way. Some with lively and concrete imagination sought to picture the elements of the material world so as to make the phenomena observed by our senses flow from the existence and movements of atoms or of corpuscles too small to be directly observed; they wanted to dismantle the machine to see all the wheels functioning. Others, more cautious and above all endowed with a more abstract mind, wanted to content themselves by uniquely representing phenomena by means of directly measurable quantities, and mistrusted the hypotheses—in their eyes too speculative and useless—of the atomists. And whereas the atomists were thus boldly advancing, opening new ways and allowing science to make astonishing progress, the energeti[ci]sts, impeded by their more formal and timid methods, retained a certain advantage from the conceptional point of view when they denounced what was simple and a little naïve in the pictures invoked by their bold rivals. But, without being aware of it, both [the atomists and the energeticists] admitted a … number of common postulates of which the future was to prove the frailty.
>
> They were, in fact, agreed in admitting the validity of the abstract framework of space and time, the possibility of following the evolution of the

physical world with the aid of quantities well located in space and varying continuously in the course of time, and the legitimacy of describing all phenomena by groups of differential equations. If the energeti[ci]sts, like Pierre Duhem, refused to allow the intervention everywhere of the 'local movement' which could be represented by a displacement of parts, they fully admitted the consideration of 'general movements' defined more abstractly by the variations of quantities in the course of time. In spite of their differences of view on the manner of carrying out this programme all theorists were then in agreement in representing the physical universe by well-defined quantities in the framework of space and time and subject to differential equations.

The differential equations … of classical mathematical physics have the common character of allowing us to follow rigorously the whole evolution of the phenomena which they describe, if we suppose that there are certain known data relative to an initial state corresponding to a particular value of time. From this there was deduced the possibility of establishing a kind of inevitable interconnexion of all the phenomena, and thus was reached the conception of a universal determinism of physical phenomena. It is not my purpose to examine from the philosophical point of view the idea of universal determinism, and I have not to ask myself, for example, if the mind, which, after all is said and done, is the creator of mathematical physics, could recover its place in a nature conceived of in such a rigid manner. What is certain is that physical phenomena, in so far as they were exactly represented by the differential equations of classical physics, were submitted to a very precisely defined determinism.

Classical physics thus represented the whole physical universe as projected with absolute precision into the framework of space and time, evolving from it according to the laws of an inexorable necessity. It completely set aside the means used to arrive at a knowledge of the different parts of this vast mechanism for, if it recognized the existence of experimental errors, it only saw in them a result of the lack of precision of our senses and of the imperfection of our techniques, and accepted the possibility of reducing them indefinitely, at least in principle, by an adequate improvement in our methods. All these representations rested essentially on the classical ideas of space and time; for a long time they appeared sufficient for a description of the evolution of the material world.[9]

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9. Louis de Broglie, *Physics and Microphysics*, trans. Martin Davidson (New York: Harper Torchbooks, 1960), 116–18. "Descartes' programme" refers, as de Broglie puts it, to "the possibility of describing natural phenomena by figures and by motion in the framework of space and time … capable of allowing, always and everywhere the establishment of rigid and precise ties of inevitable succession amongst all natural phenomena" (110). This project is clearly set out by Descartes in *World* and *Principles of Philosophy*: *Oeuvres de Descartes*, ed. Charles Adam and Paul Tannery, 11 vols (Paris: Vrin, 1996), vols VIIIA, IXB, XI, henceforth cited as AT followed by volume and page number; English translation by John Cottingham, Robert Stoothoff, and Dugald Murdoch, *The Philosophical Writings of Descartes*, 2 vols (Cambridge: Cambridge University Press, 1985), vol. I, henceforth cited as CSM followed by volume and page number. *Principles* 2.64 provides an especially compact formulation: "The only principles that I accept or require in physics are those of geometry and pure mathematics; these principles explain all natural phenomena, and enable us to

From this long passage, I distill three basic characteristics of all classical physics. Implicit therein is a notion of the relation between human mind in its knowing activity and the physical world that is to be known. The three characteristics and the associated mind–world relation stand opposed, on the one hand and in one way, to Aristotelian physics, and, on the other and in a different way, to quantum physics. The three are:

- continuity of space, time, and motion;
- spatio-temporal imageability of fundamental processes; and
- universal determinism.

Let us consider each in turn.

First, *continuity of space, time, and motion* is a shorthand formula for what de Broglie more accurately describes: continuity in the spatiotemporal variation of physical quantities such as the position, **r**(t), and velocity, **v**(t), of a discrete particle,[10] or the intensity of a continuous (say, electrical) field, **E**(**r**,t). As an apt example of what is physically meant by continuity, consider a body in motion that suddenly disappears from its present position and instantaneously reappears at a different position; this would be absurd, as in a dream. A planet, for example, does not change its distance from the sun by suddenly disappearing from its present position and instantaneously reappearing on an orbit with a different radius. The motions of planets and, more generally, the local motions of bodies, are continuous; there are no jumps. But jumps do occur among the electronic states or orbitals of atoms and molecules; it is part of their distinctive form of stability. The classical continuity of space, time, and motion thus contrasts sharply with the state transitions of quantum physics.

What about Aristotle? Does he deny continuity of local motion in favor of some sort of quantum jumps? He does not. For, in *Physics* 4.11, Aristotle says, "[s]ince a moving thing is moved from something to something and every magnitude is continuous, the motion follows the magnitude; for through the magnitude's being continuous, the motion too is continuous, and through the motion the time;" and, again, in *Physics* 6.1, "it is impossible for anything continuous to be made of indivisible things; for example, a line cannot be made of points, if the line is continuous and the point indivisible [*Meta*., 1016b27]. … And it belongs to the same argument for a magnitude, a time, and a motion to be composed of and divided into indivisibles, or none of them."[11] Finally, in *Physics* 6.2: "every

provide quite certain demonstrations regarding them." AT VIIIA, 78, CSM I, 247. "[G]eometry" here means Descartes' own analytical geometry applied to numerical magnitudes possessing physical dimensions, as described in Rule 14 of Descartes' *Rules for the Direction of the Mind* (AT X, 447–50, CSM I, 62–4), and familiar today as length, mass, time, charge, etc.

10. Or, for most applications, positions and velocities of many particles, $r_1, \ldots r_N, v_1, \ldots v_N$. As N increases, probability theory is then brought to bear, e.g., in kinetic theory of gases and statistical mechanics.

11. *Physics* 4.11, 219a11–13, 6.1, 231a24–5, b18–20; trans. Joe Sachs, *Aristotle's* Physics*: A Guided Study* (New Brunswick, NJ: Rutgers University Press, 1995), 121, 147–8.

magnitude is divisible into magnitudes (for it has been shown that it is impossible for anything continuous to be made of uncut-able parts, and every magnitude is continuous)."[12] Therefore, the opposition between classical physics and that of Aristotle concerning space, time, and motion must be on a different level, namely, that of the relation between human mind in its knowing activity and the physical world that is to be known.

What must be appreciated in de Broglie's exemplary expression of the self-understanding of classical physics is that mind is there looking not directly at the world but rather at the products of its own constructive activity: elaborate structures of real-numerical variable magnitudes of which the variables, x, y, z, and constants, a, b, c, etc., of Descartes' *Geometry*, the source of our analytic geometry, are the prototype. The material world external to this sophisticated mental artifice is admitted as *known* only guardedly, under the carefully controlled conditions of the experiment: does the numerical result of the empirical measurement *match* the mathematically predicted numerical value to within current limits of experimental precision (the error intervals)? If so, then the conceptual structure of variable quantities possessing physical dimensions, the equations and graphs derived from the laws of physics, that is, the *model* that has been proposed by us to nature is accepted as corroborated.[13] If not, then more adequate models based on revised equations or new laws are to be constructed; we are on our way into the standard philosophy of hypothetico-deductive science that is available in many textbooks. For our purposes here, let us note that the mind–world relation characteristic of physics has two components, the mathematical model and the extra-mathematical reality to be modeled: they "transact" in the "common currency" of number, calculated and measured.

The world-conception of classical physics described by de Broglie assumes the progressive improvement of the numerical precision of experimental measurements, and thus of the match between filtering mind and the filtered ("true") world. It thus assumes the complete adequacy of real number to nature. The point for now is simply that this is not the type of mind–world relation that is found in Aristotle's *Physics*. There, intellect looks directly through the senses at the kinds of beings that make up the world: the elements (the land, the waters, the weather), plants, animals, artifacts and the human beings that produce them, and up, above

12. *Physics* 6.2, 232a23–5; Sachs, 149.

13. "[M]athematical physics … is bent on *matching* the consequences derived mathematically from hypotheses with observations dictated by these hypotheses … It is not its business to say what, for example, gravitation, or electromagnetism, or energy *is*, except by establishing in a symbolic-mathematical formula the relations that bind these entities (if it is at all permissible to use this word) to observable and mathematically describable magnitudes." Klein, "On Precision," in *Lectures and Essays*, 289–308, here 305–6. And, above all, Kant: "reason has insight only into that which it produces after a plan of its own, and … it must not allow itself to be kept, as it were, in nature's leading-strings but must itself show the way with principles of judgment based upon fixed laws, constraining nature to give answer to questions of reasons own determining." Immanuel Kant, *Critique of Pure Reason*, B xiii, trans. Norman Kemp Smith (New York: St Martin's Press, 1965), 20.

it all, the celestial bodies, which appear to move so differently from all the rest. Each of these beings is a whole whose parts are related in various ways, and all are subject to being moved, to acting and being acted upon, in various ways according to the causes judged to be necessary in order to fit speech to the phenomena, beginning with ourselves who bespeak the phenomena.[14] Aristotle's way of thinking together the objects of his science of nature is succinctly indicated at *Physics* 3.1, 200b32–4: "what moves is a mover of something moved, and what is moved is moved by something moving it, and there is no motion apart from things [οὐκ ἔστι δέ τις κίνησις παρὰ τὰ πράγματα]"[15]—motion, mobile, causes of motion must be thought together, along with the time and (in local motion, increase and decrease) the magnitude traversed. Finally, for Aristotle, the meaning of physical motion (κίνησις) cannot be adequately expressed without using the words most characteristic of his understanding of nature: act and potency, ἐνέργεια and δύναμις:

> motion is the actuality of the potentially being as such (ἡ τοῦ δυνάμει ὄντος ἐντελέχεια ᾗ τοιοῦτον κίνησις ἐστιν). … it cannot be placed in an unqualified way either under the potentiality or under the actuality of things (οὔτε εἰς δύναμιν τῶν ὄντων οὔτε εἰς ἐνέργειαν). … a motion [then] is sort of an actuality (ἐνέργειαν μέν τινα εἶναι) … such as we have stated, difficult to grasp but capable of existing (χαλεπὴν μὲν ἰδεῖν ἐνδεχομένην δ'εἶναι).[16]

This is far from the representation of motion as position, x(t), velocity, v(t) = dx/dt, etc. There, the symbols, x, v, t, dx, dt, possess a kind of objectivity, or being in their own right, deriving sense or meaning through their membership in the system of signs constituted by the axioms and binary operations of algebra prior to, and independently of, any possible application to the material and mobile beings that we see with our eyes and point at with our fingers.[17] For us, there *is* motion—mathematical "motion"—apart from things.[18]

14. See especially *Parts of Animals* 1.1, 639a1–41a18, for the self-referential character of Aristotle's natural philosophy.

15. *Phys.* 3.1, 200b32–4; Sachs, 73. The passage goes on to say that there can be no change or motion outside the categories of substance, quantity, quality or place. See also *Phys.* 2.2, 193b23–194a15 and *Meta*. 6.1, 1025b29–26a4: mathematical objects are separable from motion (κίνησις) and matter, but motion is not separable from matter.

16. *Phys*. 3.1, 201a11–12, 3.2, 201b29–30, 202a2.

17. "[B]ecause the mere perceptual content of the signs involved in symbolic mathematics [e.g., 2, x] is insufficient to establish their mathematical significance, this significance must somehow be stipulated by the calculative method employed [the rule-governed method of designating and manipulating sense perceptible signs to perform 'calculations' with general mathematical objects], only after which the sense-perceptible signs can become known as 'symbols'." Burt C. Hopkins, *The Origin of the Logic of Symbolic Mathematics: Edmund Husserl and Jacob Klein* (Bloomington, IN: Indiana University Press, 2011), §199, 491. An excellent summary of algebra for physicists is provided by Richard P. Feynman, Robert B. Leighton, and Matthew Sands, *The Feynman Lectures on Physics* (Reading, MA: Addison-Wesley, 1963), vol. 1, chap. 22.

18. "We commence with a chapter on *Motion*, a subject totally independent of the existence of *Matter* and *Force*." William Thomson and Peter Guthrie Tait, *Treatise on Natural Philosophy* (Oxford:

Being detached or separate from material being, the mental apparatus of our formal-symbolic mathematics is like a universal and demiurgic tool that, as such, is separate from (not bound into, not a part of) the material to be worked on and, therefore, is available in advance for application to any material. In his review of Klein's *Lectures and Essays*, with its emphasis on the Greek understanding of number, *arithmos*, Richard Kennington provided the following apt formulation:

> [T]he Cartesian *numerus*, in contrast to the *arithmos*, cannot function as the bond of our world-relatedness; or rather its lack of such relatedness permits what is other than man to be understood as 'world' and Cartesian man to be world-less.[19]

It is Plato for whom number is the bond of our world-relatedness; for Aristotle, it is soul (e.g., *De Anima* 3.5, 429a27, 3.8, 431b21). But of course Descartes transforms soul, too.

A sign of the difference between Aristotle's physics and classical physics is de Broglie's reference to "coordinates [x, y, z, t] … the abstract framework [the classical ideas] of space and time." As Klein states it at the conclusion of his account of the concept of number in Descartes:

> [I]n Descartes' thinking, the dignity of representing the substantial 'being' of the corporeal world accrues to extension precisely by reason of its *symbolic* objectivity within the framework of the *mathesis universalis*. Only at this point [Descartes' invention of general magnitude as the fundamental term of physics] has the conceptual basis of 'classical' physics, which has since been called 'Euclidean space', been created. This is the foundation on which Newton will raise the structure of his mathematical science of nature.[20]

This concludes my comment on the first of the three basic characteristics of classical physics, continuity of space, time and motion, with comparison and contrast to quantum physics and Aristotelian physics.

Clarendon Press, 1857), v. Thomson and Tait's *Treatise* was a standard textbook of physics in the nineteenth century. A contemporary textbook is that of David Halliday and Robert Resnick, *Physics* (New York: John Wiley & Sons, 1978), 30: "Mechanics, the oldest of the physical sciences, is the study of the motion of objects. … When we describe motion we are dealing with that part of mechanics called *kinematics*. When we relate motion to the forces associated with it and to the properties of the moving objects, we are dealing with *dynamics*."

19. Richard Kennington, review of *Jacob Klein Lectures and Essays*, *Review of Metaphysics* 41, no. 1 (September 1987), 144–9, here 147.

20. Klein, *GMTOA*, 211. The quotation marks indicate that "Euclidean space" is inaccurately so called in the sense that Euclidean geometry is not coordinate (numerical) geometry—that is the achievement of Descartes (and Fermat). See Klein, "World of Physics," *Lectures and Essays*, 21. In post-Newtonian physics, however, "Euclidean space" is often used in contradistinction to coordinate spaces with other metrics or distance functions, e.g., the Minkowski space of special relativity and the Riemannian space of general relativity. Descartes' coordinatization of geometry thus opens doors to remarkable physical theories whose development would be difficult to imagine without the enabling Cartesian foundation.

The second basic characteristic described by de Broglie is *spatio-temporal imageability of fundamental processes.* It means that we can always project in our mathematical imagination and express on paper Cartesian spatial coordinate axes, x, y, z, and then picture the relevant physical quantities—magnitudes with dimensions of length, mass, charge, time, and their combinations—varying in that space with time, t. In particular, we can know, from the data and the model, what is going on *inside* any physical system—an atom, an organism, a laboratory (e.g., two-slit interference) apparatus[21]—even though the mind is looking mainly at the products of its own activity! We can imagine, for example, a particle with a precise position given by its Cartesian coordinates, x(t), y(t), z(t), and precise velocity with components, $v_x(t)$, $v_y(t)$, $v_z(t)$, for any time, t, moving on what is thereby defined as its *trajectory*. Or we can imagine (slightly more abstractly) the intensity of an electric field, **E**(x, y, z, t), varying in both spatial position, x, y, z, and time, t. This way of using the mind is assumed to be fully adequate to the nature of things; the common currency of number or, more precisely, numerical magnitude is taken as sufficient for all transactions between the mathematics (equations and calculation) and the physics (measurement and matching). This assumption does not entail the rather crude Cartesian *identification* of physical objects with mathematical objects,[22] but rather the more sophisticated assumption that, whatever may be the differences between mathematical objects and physical objects, such differences can make no difference for the conduct of our (now thoroughly mathematical) physics. In other words, the natural-philosophic or metaphysical question of the difference between mathematical objects and physical objects can be suspended; let it be a private matter. This is what I mean, following De Gandt, by "physico-mathematical secularism."

The third basic characteristic described by de Broglie, *universal determinism*, means that, through the equations of motion, the numerical values of the relevant quantities at one instant of time, t, or position, x, in space enable us to calculate the values of those quantities at the next instant of time, t + Δt, or adjacent position in space, x + Δx, and the next, on into future time and distant space. No other type of causality, beyond initial data or boundary conditions and equations of motion, is needed to account for all natural phenomena. But, as should now be clear, this determinism—first made explicit by Laplace[23]—is based in the mathematics (i.e.,

21. De Broglie's description of the atomists is apt: "they wanted to dismantle the machine to see all the wheels functioning." With the turn of the nineteenth century, the more phenomenological and cautious energeticists (Mach, Duhem, Poincaré, Ostwald) were vanquished by the rapidly mounting body of molecular theory corroborated by experimental evidence, e.g., Einstein on Brownian motion. See Jean Perrin, *Les Atomes* (Paris: Librarie Felix Alcan, 1913).

22. "I conceive its [matter's] extension … not as an accident, but as its true form and essence" (Descartes, *The World*, Chap. 6, AT XI, 36, CSM I, 92). "I recognize no matter in corporeal things apart from that which the geometers call quantity" (*Principles* 2.64, AT VIIIA, 78–9, CSM I, 247). See also *Principles* 4.187, AT VIIIA, 314–15, CSM I, 279.

23. Pierre Simon Laplace, *A Philosophical Essay on Probabilities*, trans. F. W. Truscott and F. L. Emory (New York: Dover, 1951), 4, 6.

it is a feature of the mathematical description). Its alleged universality in physics is a philosophical claim inspired by particular instances of success, to wit, *to the extent that* the behavior of a physical system (e.g., the solar system) is found by observation or experiment to match the mathematical predictions of the model to within given limits of precision, we say that the physical system thereby modeled is deterministic. The successes of classical physics on various classes of deterministic phenomena (celestial mechanics, electromagnetism, thermodynamics, vibrations and waves in material media) were stunning, but they were always empirical and particular or partial. And so the claim for *universal* determinism (of *all* natural phenomena) was a matter of the disposition and imagination of scientists and philosophers, not scientific warrant.

To state essential conclusions thus far: Within the self-understanding of classical physics, strong claims (for continuity, imageability, determinism) about the transparency and the malleability of nature are made based, first, on the match (in particular cases) between the mind's models and empirical measurements, and second, on a disposition to ascend from particular scientific results to universal philosophic claims.

Let us illustrate the classical conception of the physical world by reviewing the basic logic of the Newtonian calculation of the trajectory of a body moved under gravitational force. This will exemplify de Broglie's account, and sharpen the comparison to Aristotle's physics, on the one hand, and quantum physics, on the other.

2 Essential Logic of the Newtonian Trajectory Calculation

My aim here is to enable us to *see the reason* for Newton's key achievement: the predictive calculation of the trajectory of a body in motion under gravitational force, e.g., a planet or a comet around the sun. To this end, I follow, not the current textbook approach, but the more insightful description of Einstein and Infeld in *The Evolution of Physics*, a book written for non-specialists.[24]

The problem to be solved is this: Given by empirical observation the position, $\mathbf{r}_0$, and velocity, $\mathbf{v}_0$, of a planet or comet relative to the sun at a given initial time, t_0, to derive the position and velocity, $\mathbf{r}_1$, $\mathbf{v}_1$, at a later time, t_1, without further recourse to observation.[25] The principles governing the calculation are two: Newton's second law of motion, and his law of universal gravitational force. We must examine the role played by each and recognize the crucial significance of the gravitational force law.

24. Albert Einstein and Leopold Infeld, *The Evolution of Physics* (New York: Simon and Schuster, 1968), 9–30.

25. We use the standard vector notation for position and velocity in Cartesian coordinates, x, y, z, and time, t: $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$, and $\mathbf{v}(t) = (dx/dt)\mathbf{i} + (dy/dt)\mathbf{j} + (dz/dt)\mathbf{k}$, where $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ are unit vectors along the x, y, z axes. The initial position and velocity, $\mathbf{r}(t_0) \equiv \mathbf{r}_0$, $\mathbf{v}(t_0) \equiv \mathbf{v}_0$, are the "known data relative to an initial state corresponding to a particular [initial] value of time [t_0]" referred to by de Broglie.

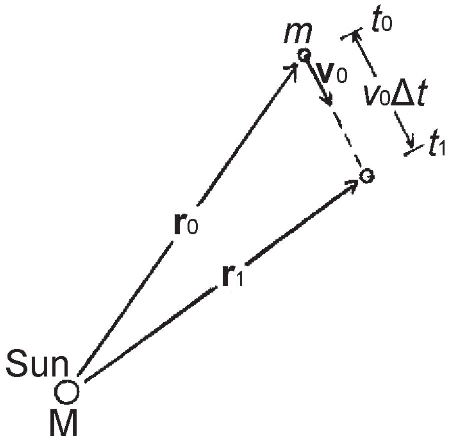

Figure 1 Constructing $\mathbf{r}_1$

The conceptual framework of space is, in this case, the two-dimensional plane with the sun, of mass M, represented at the origin of coordinates.[26] We represent the initial position and velocity vectors, $\mathbf{r}_0$, $\mathbf{v}_0$, of the planet or comet, of mass m, as shown in Figure 1. Can anything at all be predicted from our two pieces of initial data? The answer is, yes, approximately.

Knowing $\mathbf{v}_0$, the initial velocity, we can *estimate* the position, $\mathbf{r}_1$, of the planet or comet a short time later, at time t_1, by taking its motion as rectilinear (even though it is necessarily curvilinear due to the gravitational force of the sun). This is erroneous, but we can make the error as small as we like by making the time interval, $\Delta t = t_1 - t_0$, smaller and smaller. So we "move" the planet or comet, that is, we move the point that represents it in our figure, in the direction of v_0 a distance that represents $v_0\Delta t$ in the solar system. For example, if we are studying the motion of Mars, whose mean orbital speed is 24.1 km/sec, then, picking $\Delta t = 1$ sec, we would plot its new position at a distance in the direction of v_0 representing 24.1 km in space from its initial position. We have now derived from the initial data, r_0, v_0, the approximate position, r_1, of the body at time t_1.

But what is the new velocity, $\mathbf{v}_1$? As noted, a planet or comet under the gravitational attraction of the sun moves on a curved path (it must, by Newton's first law). Thus the direction of the velocity vector that we use to represent its motion is constantly changing. To estimate the new velocity, $\mathbf{v}_1$, at time t_1, we must find the change in velocity of the body, $\Delta\mathbf{v}_{01}$, that occurs during the first time interval, Δt, from t_0 to t_1. Then we can construct, by vector addition, $\mathbf{v}_1 = \mathbf{v}_0 + \Delta\mathbf{v}_{01}$. This would complete our prediction of the new position *and* velocity of the planet or comet a short time, Δt, into the future. This may seem like a small step, because

26. Angular momentum, $\mathbf{L} = \mathbf{r} \times m\mathbf{v}$, is conserved in the motion of a body of mass, m, under any central force, thus for an inverse-square gravitational force. Therefore, the position, $\mathbf{r}$, and the velocity, $\mathbf{v}$, of the moved body must lie in a fixed plane. I am assuming the masses are known—a big assumption (how did Newton figure it out?)—in order to focus on the most essential point.

Δt is small (e.g., one second), but if we can repeat this procedure for the next time interval, from t_1 to t_2, deriving $\mathbf{r}_2$ and $\mathbf{v}_2$ from $\mathbf{r}_1$ and $\mathbf{v}_1$, and so on, then we will have made a world-historic revolution in human thought. The focus of our attention, therefore, is on $\Delta\mathbf{v}_{01}$: how is it determined? This is where Newton's law of universal gravitation comes into play; it is the crucial link in the logic of the trajectory calculation—linking $\Delta\mathbf{v}_{01}$ to $\mathbf{r}_0$—without which our procedure would stop dead after the estimate of $\mathbf{r}_1$.

The second law of motion relates the net force on a body of mass m to its resulting acceleration or time rate of change of velocity: $\mathbf{F} = m\Delta\mathbf{v}/\Delta t$, in the limit as $\Delta t \rightarrow 0$. But what *is* the net force on the planet or comet under study? It is the law of gravitation that relates the known distance, r_0, to the net force exerted by the sun on the planet or comet at time t_0. In present-day notation,[27] $\mathbf{F}(t_0) = -\mathbf{r}_0 GMm/r_0^3$. Thus combining the law of gravitation with the second law of motion enables the determination of the (approximate) change in velocity, $\Delta\mathbf{v}_{01}$. The direction of $\Delta\mathbf{v}_{01}$ is that of the force, toward the sun, and its magnitude is calculated as follows:

$$-GMm/r_0^2 = m\Delta v_{01}/\Delta t$$

Therefore, $\Delta v_{01} = -GM\Delta t/r_0^2$, in which all the quantities on the right hand side are known. Having thus determined $\Delta\mathbf{v}_{01}$, we have by vector addition, $\mathbf{v}_1 = \mathbf{v}_0 + \Delta\mathbf{v}_{01}$. The last step in the calculational procedure is to transfer ("move") the vector $\mathbf{v}_1$ parallel to itself (preserving both its magnitude and its direction) from position $\mathbf{r}_0$ to position $\mathbf{r}_1$ (Figure 2).

We have completed our task: we have derived $\mathbf{r}_1$, $\mathbf{v}_1$ from $\mathbf{r}_0$, $\mathbf{v}_0$. We can repeat this algorithm: From $F_1 = -GMm/r_1^2$ derive $\Delta\mathbf{v}_{12}$, and then construct $\mathbf{v}_2 = \mathbf{v}_1 + \Delta\mathbf{v}_{12}$, plotting $\mathbf{r}_2$, $\mathbf{v}_2$, at time t_2, etc. *Thereby we trace out the orbit or trajectory of the body* moving under the gravitational force of the sun, without further recourse to observational data. After performing the calculations, we ask, does the prediction match future observation? For example, does the planet Mars or Halley's comet in fact appear in the night sky when and where we predicted it would? Within the limits of observational precision, yes, it does.

In this account of the concept of *trajectory*, I have suppressed technical details (energy and angular-momentum considerations, non-linearity, the many-body problem, and calculus) in order to display what is essential to the kind of thinking characteristic of classical physics. Note especially the last step of the above procedure (in Figure 2): the velocity vector (defined by its magnitude, i.e., speed, and its direction) can be detached from one position, $\mathbf{r}_0$ (where we performed the

27. The radial direction of the force (toward the sun) is expressed by the vector, $\mathbf{r}_0/r_0$, of unit length. For a discussion of Newton's own distinctive kinematic-infinitesimal geometry in relation to the analytic-algebraic mathematics that quickly followed see my "The Exemplary Career of Newton's Mathematics," *The St John's Review* 44, no. 1 (1997), 73–93, also in PDF from http://philosophy.cua.edu/Faculty/rfh. The differences between these two types of mathematics are not relevant for present purposes as explained in note 60, below, on the symbolic character of both Descartes' and Newton's magnitudes.

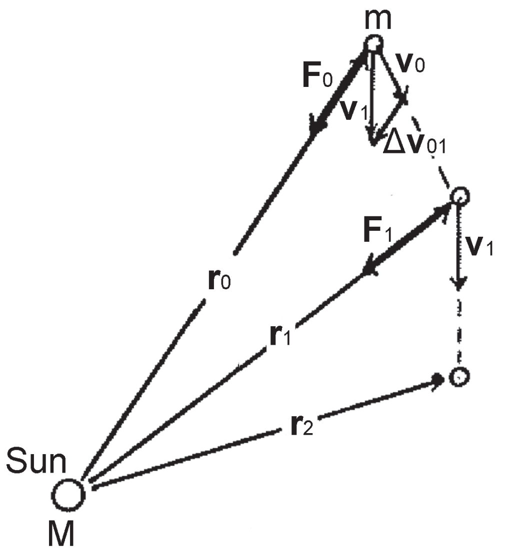

Figure 2 Constructing $\mathbf{v}_1$

vector sum, $\mathbf{v}_0 + \Delta\mathbf{v}_{01}$), and "moved" on our plotting paper (always parallel to itself to keep it the same vector) to another position, $\mathbf{r}_1$. We can *reify* motion and then manipulate it to suit our problem-solving purposes. This is vivid testimony to our ability to conceive "motion apart from things" and our possession of an ingenious *art* of solving certain problems of *natural* local motion and force. Let us note, however, that this physico-mathematical art knows neither of Aristotle's act and potency, nor of Heisenberg's uncertainty principle.

3 Species-neutrality, or Univocalizing the Analogical

I touch on the fundamental feature of species-neutrality insofar as it makes contact with Klein's work.

Normally, the way a body moves or behaves is intimately related to what kind or species of body it is, as known through ordinary sense perception. Pigs don't fly, sparrows don't oink. Accordingly, the way in which two bodies interact (how they attract and/or repel each other) depends on what kind or species each is. The traditional, Aristotelian and Scholastic name for the essential relation between the source of activity (form) and the supporting structure (matter) in a body is hylomorphism.[28] Quite generally, in Aristotle's science of nature, the intelligible

28. Hylomorphism is reflected in assertions like the following in Aristotle: "[M]atter is among the relative things: for a different form, a different material." *Phys.* 2.2, 194b9, Sachs, 53. And: "[A]ll things that change have material but different sorts of material." *Meta.* 12.2, 1069b25, trans. Joe Sachs, *Aristotle's* Metaphysics (Santa Fe, NM: Green Lion Press, 1999), 232. See also *Phys.* 8.1, 251a12–13.

principles—natural form and common sensible matter—that account for the behavior of the sensed particulars (a stone, a pig, a sparrow, a human being) are species-typical or species-specific. Form, matter, and privation are indeed universal principles of natural things, but they are predicated of the different kinds (e.g., elements, living things, human beings, celestial bodies) analogically, not univocally:

> [I]t is not possible to say … that *all* things have the same elements and principles, except by analogy, just as if one were to say that there are three kinds of principles: form, privation, and material. But each of these is different as it concerns each class of things.[29]

Newton's law of gravitation does not exemplify the Aristotelian, species-specific type of relation between sensible effects and intelligible principles. The algebraic equation, $F = -GMm/r^2$, expresses an intelligible principle of local motion in nature that is indifferent or neutral to the kind or species, the size, shape, internal structure and function of the two interacting bodies. Newton's gravitational law is thus species-neutral, as are the terms ('mass', 'distance') of which it is composed. For all bodies—celestial and terrestrial, natural and artificial, living and non-living—possess mass, m, and relative position, $\mathbf{r}$, also velocity, $\mathbf{v}$, acceleration, $\mathbf{a}$, momentum, $\mathbf{p} = m\mathbf{v}$, kinetic energy, $T = p^2/2m$.[30] The completely species-neutral universality of Newton's physics means that there is no essential difference or heterogeneity of celestial and terrestrial matter.[31] The Scientific Revolution thereby corrects the most embarrassing error of the Aristotelian doctrine of nature.

All the algebraic terms of classical mechanics are species-neutral. Here we make significant contact with Klein.

In the equation $F = -GMm/r^2$, the symbols, M, m, r—the masses and distance between the points at which the masses are taken to be concentrated—are understood univocally, not, like form and matter, analogically. The algebraicization of science entails the homogenization of our thoughts and therewith a certain detachment of the resulting concepts from things. Klein describes this in chapter 9 of *GMTOA*:

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29. *Meta*. 12.4, 1070b17–20, Sachs, 235, translation slightly modified (emphasis Sachs'), also *Meta*. 9.6, 1048a37-b10, on the analogical sense of *energeia*. The term 'body' is thus, for Aristotle, not univocal, as it is for us, but analogical.

30. Spinoza expresses succinctly the sense of the common (species-neutral) properties of what will become classical physics: "That which is common to all [bodies] … and which is equally in the part as in the whole [e.g., Cartesian extension, Newtonian mass], does not constitute the essence [the Aristotelian natural form; *Meta*. 1030a12] of any one particular thing. … Those things that are common to all … can be conceived only adequately." *Ethics*, II.37 and 38, in *Baruch Spinoza: Ethics, Treatise on the Emendation of the Intellect, and Selected Letters*, trans. Samuel Shirley (Indianapolis, IN: Hackett, 1992), 87.

31. In light of Newton's laws of motion and gravitational force, humanly controlled space flight is discovered to be possible, to be within our power. This is shown at the end of the *Principia*; see *Sir Isaac Newton's Mathematical Principles of Natural Philosophy*, trans. Andrew Motte and Florian Cajori (New York: Greenwood Press, 1969), 551; henceforth cited as *Principia*, followed by page number.

> [In the] 'new' science … [n]othing but the internal connection of all the concepts, their mutual relatedness, their subordination to the total edifice of science, determines for each of them a *univocal* (*eindeutig*) sense and makes accessible to the understanding their only relevant, specifically scientific content. … Thus every one of the newly obtained concepts [e.g., quantity, body, mass, motion, velocity, acceleration, momentum, force, work, energy] is determined by *reflection on the total context of that concept*. Every concept of the 'new' science belongs to a new conceptual dimension. The special intentionality of each such concept is no longer a problem: it is indifferently the same for all concepts; it is a medium beyond reflection (*sie ist das allgemeine, von der Reflexion nicht mehr erreichte Medium*), in which the development of the scientific world takes place.[32]

Klein speaks here of the new concepts as being determined by reflection on their "internal connection … their mutual relatedness," namely, according to those axioms and operations of algebra, then according to the physical dimensions that they bear (e.g., mass, length, time, charge) and the physical laws that they express, and, finally, an external connection, according to the operations of measurement (e.g., our use of scales, meter sticks, clocks) whereby they can be specified as, say, 10.1 kilograms, 324.8 meters, 13.6 seconds. This is a new conceptual medium beyond the old (ancient and medieval) type of reflection on the "special intentionality" of each word. For example, the word "quantity," Greek τὸ ποσόν, Latin *quantum*, thus, more accurately (because more concretely), "the quantified" or "quantified," which expressions point beyond themselves and outside our thought to the quantified things, cannot be said univocally of the two kinds, discrete number, ἀριθμός, and continuous magnitude, μέγεθος, but only analogically. These two kinds of the quantified fall in one and the same category of being because they are both divisible into parts in such a way that both accept the equal and unequal (greater than, less than); but, whereas magnitude is infinitely divisible into parts having boundaries that can touch, number is finitely divisible into indivisibles—the units—lacking boundaries that can touch.[33]

This old type of reflection, on the ways "our thought, and also our words, signify or intend their [different kinds] of objects,"[34] would preclude the homogenization of the heterogeneity of discrete and continuous into the one univocal concept of the arithmetical continuum, or the real number system. Similarly for the old *versus* the new reflection on the words "body" (σώμα) and "motion" (κίνησις), and so forth.[35]

32. Klein, *GMTOA*, 120–21.

33. Aristotle, *Meta.* 5.13, *Cat.* 6. The even and the odd belong to number, but not to magnitude. A square magnitude can always be divided into two equal square magnitudes, but a square number can never be divided into two equal square numbers.

34. Eva Brann (translator), *GMTOA*, 118.

35. In "World of Physics," Klein states that in fact the late Scholastics had drifted into this new conceptual medium without knowing it and thus without knowing how to use it to maximum effect, which involves turning the mind in a new direction and questioning in a new way (*Lectures and Essays*, 3, 6–7).

We have gone from the species-neutrality of classical physics, to the univocalization of analogies, and homogenization of heterogeneities, in both mathematics and physics. We thus return—armed with more evidence—to an important result: mind's detachment from the world through the algebraicization of thinking.

4 Reductionist Generalization: Newton's Universal Forces-and-Particles Model

In the Preface to the *Principia*, Newton generalizes from his particular gravitational theory to the universal forces-and-particles model, a mental image of everything physical in the whole universe, and a program for future research:

> I derive from the celestial phenomena the forces of gravity with which bodies tend to the sun and the several planets. Then from these forces … I deduce the motions of the planets, the comets, the moon, and the sea. I wish we could derive the rest of the phenomena of Nature by the same kind of reasoning from mechanical principles, for I am induced by many reasons to suspect that they may all [!] depend upon certain forces by which the particles of bodies, by some causes hitherto unknown, are either mutually impelled towards one another, and cohere … or are repelled and recede. … These forces being unknown, philosophers have hitherto attempted the search of Nature in vain.[36]

Here we have a grand analogy but it is not of ancient and medieval type (conveying ontological sameness and difference). Rather, it is conceptually homogeneous, as in "big circles are analogous to small circles"—to wit, every body is like a solar system writ small. Here, all the sensible, composite bodies are mentally conceived as clouds of subsensible particles, which move in space on in-principle calculable trajectories. It is assumed here that the intelligible principles of natural phenomena will, like the gravitational force law, be expressible in species-neutral terms like mass, and the spatial relations of particles, point-like centers of attraction and repulsion. The discovery of electric charge and Coulomb's law of electrical attraction and repulsion, similar in its algebraic form to Newton's law of gravitation, gave the Newtonian program great impetus. Thus in 1847 Helmholtz proclaimed the goal of physical science as the complete intellectual penetration of nature by human mind:

> [N]atural phenomena are to be related to the motions of matter possessing unchanging forces of motion, which forces depend only on spatial relations. … The force, however, which two whole masses exert on each other must be resolved into the forces of all their parts on one another; thereby mechanics goes back to the forces of material points. … Finally, then, the task of the physical natural sciences is specified thus: to reduce natural phenomena to

36. Newton, *Principles*, 1686 Preface, xviii.

> unchanging attractive and repulsive forces, whose strength depends on the distance. The realizability of this task is, at the same time, the condition of the complete comprehensibility of nature.[37]

De Broglie's general description is here, in the words of Newton and Helmholtz, exemplified for "the mechanical view." According to this world-conception, all the properties and activities of all the wholes in nature are derivable from, or entailed by, the local motions and quantitative properties of their constituent particles. Thus mass, charge, associated force laws, particle position, velocity, momentum, energy, associated densities, distributions, and flows are taken to be the terms adequate for the explanation of all natural phenomena.

What does the forces-and-particles model of the universe imply, according to its own inner logic, about nature as we prescientifically encounter it, specifically, as articulated into visible kinds or species (e.g., "lion, eagle, rose, gold, and the like")?[38] The following paradoxical implications clash with our ordinary sense-perception-based experience of, and belief about natural things, both living (a lion, an eagle, a rose) and non-living (gold), but especially about living things.

Wholes are reducible to sufficiently simple parts or particles; parts are prior to wholes, ontologically and thus epistemologically. The ontological priority of parts to wholes means, in the language of Aristotle, that parts are fully actual in the whole, as in the case of artifacts. The alternative possibility—that there exist wholes irreducible to their parts or holistic systems in nature—would require that the particles of such wholes, and the force laws by which the particles interact, would have to be somehow affected or modified or limited in their being by their membership in that kind of whole. But that possibility—that there might be parts potential and not fully actual in the whole—is rejected a priori by Newton's forces-and-particles model. The locus of that rejection is the parallelogram rule for composition of forces, Corollary II of the *Principia*.[39] Wholes that, unlike artifacts, strike our senses as irreducible to parts (they seem to be more than aggregates) are the ones we call "alive," because of their intrinsic unity (if we try to pull an animal apart it bites and scratches, works hard to keep itself together) and characteristic stability of their kind (cats have kittens, dogs have puppies). Biological phenomena, of course, fuel Aristotle's account of form as holistic principle.

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37. Helmholtz, "On the Conservation of Force," *Wissenschaftliche Abhandlungen* (Leipzig, 1882), vol. I, 15–16; translation mine.

38. Bacon, *New Organon* II.17, Anderson, 152. Bacon is well aware of the dualism of subsensible particles and laws, on the one hand, and sense-perceptible compounds, on the other—a dualism of "the homogeneity of laws and the heterogeneity of kinds," in Thomas Prufer's apt phrase—and thus of the need to explain how the former give rise to the later. But no such explanation is provided in the *New Organon*.

39. In *Principia*, Corollary II, the conjoint action of the particles composing a compound body is assumed to be the sum of the actions (forces) of each particle taken separately. See my "Wholes, Parts, and Laws of Motion," *Nature and System* 6 (1984), 195–215, and "Animals versus the Laws of Inertia," *Review of Metaphysics* 46 (1992), 29–61, also in PDF from http://philosophy.cua.edu/Faculty/rfh.

Reductionism and species-neutrality imply that nature is malleable; that our power to change things is much greater than pre-modern science imagined. For, if the intelligible principles (the properties of the particles and the force laws whereby they interact) are species-neutral, then the heterogeneity of species, so evident to our senses, must not result *per se* from those intelligible principles, but must rather be, in some surprising way, accidental. That is: the sensible species are not the effects of causes aimed *per se* at those effects (like Aristotelian forms), and so their differences (the difference between pigs and sparrows) are not rooted in the essential nature of things, and, furthermore, might not be a barrier to our operation. After all, the overthrow of the erroneous doctrine of the essential difference between celestial and terrestrial matter revealed the possibility of humanly controlled space flight.

Finally, the forces-and-particles model implies that there are no privileged moments or states, no ends in nature. A configuration of particles moves under the equations of motion as determined by initial data (e.g., particle positions and velocities) at any given time.[40] There is no "room" in the basic logic of the trajectory calculation for a future state to be a cause of present motion, e.g., like healing directed to the restoration of health, or, as we shall see, an atom returning to its ground state after an external disturbance.

Let us turn 180 degrees for a further look at the Aristotelian alternative, an understanding of nature that is species-specific and holistic, and accordingly less technologically potent.

5 Aristotle: Form prior to Matter, Wholes prior to Parts

Phys. 2.1 provides the argument for form as principle and cause of change and stability in a natural substance:

> The things existing by nature [not by art] all appear to have *within themselves* a principle of motion and rest. … [So] nature is a certain principle and cause of being moved and of coming to rest *in* that to which it belongs, primarily and essentially and not accidentally.[41]

40. "We ought then to regard the present state of the universe as the effect of its anterior state and as the cause of the one which is to follow. … The regularity which astronomy shows us in the movements of the comets doubtless exists in all phenomena. … the only difference is that which comes from our ignorance" of the present positions and velocities of all the particles and the forces by which they interact; the difference does not come from the natures of things. Laplace, *Probabilities*, 6.

41. Τὰ μὲν γὰρ φύσει ὄντα πάντα φαίνεται ἔχοντα ἐν ἑαυτοῖς ἀρχὴν κινήσεως καὶ στάσεως … ὡς οὔσης τῆς φύσεως ἀρχῆς τινος καὶ αἰτίας τοῦ κινεῖσθαι καὶ ἠρεμεῖν ἐν ᾧ ὑπάρχει πρώτως καθ’ αὑτὸ καὶ μή κατὰ συμβεβηκός. *Physics* 2.1, 192b14–15, 21–3; emphasis mine. The *appearance* of having—unlike artifacts—an irreducibly internal source of motion is accepted by Aristotle as reliable. The higher animals and our connaturality to them (e.g., they have faces, organs of perception, they voice pain and pleasure, like us) provide the most persuasive examples.

Nature is both form and matter, and the "form is nature more than the matter."[42] The complicated adverbial phrase "primarily and essentially and not accidentally" means (among other things) that natural form is internal to the moved thing (e.g., an animal) in a way that cannot be fully derived from, or completely reduced to its material parts: "[Natural] things will be neither without material nor determined by their material."[43] Form is thus a holistic principle; the parts of a naturally informed compound are what they are and act as they do only in terms of the whole they compose. If separated by dissection from the whole, they cease to be what they were:

> [T]he whole must of necessity be prior to the part; for if the whole [body] is destroyed there will not be a foot or a hand, except in the sense that the term is similar (as when one speaks of a hand made of stone). … For it is not a hand of any sort that is part of a human being, but only one capable of accomplishing its work, and therefore being ensouled; if not, it is not a part of it.[44]

Therefore, one whole informed material substance (e.g., a squirrel) cannot be adequately understood in terms of its parts. Rather, since the whole is ontologically prior to the parts, the parts cede some of their being to the unifying authority of the form. Thus, for Aristotle, the parts of a natural substance exist only potentially (δυνάμει) in the whole; they are not fully actual in the whole:

> [W]hat is continuous and limited is a whole whenever some one thing is made of a plurality of things, most of all when they are distinct constituents of it only potentially, but if not, actually. … [W]henever the parts [of an animal] are one and continuous by nature … they will exist potentially.[45]

In fundamental contrast, the parts of an artifact (e.g., a clock) are what they are independently of the whole; they are fully actual in, and thus in their being, are neutral or indifferent to the whole.

Aristotle's doctrine of parts being potentially in the whole is admittedly obscure. We can get a sense of it only by consideration of his examples, which are almost all biological.[46] In any case, despite the obscurities, it is fair to say that, within this understanding, the form is not only a source but also a limit to our knowledge and control of nature:

> Limit means … the substance of each thing, and the essence (τὸ τί ἦν εἶναι) of each thing; for the latter is a boundary of knowledge, and if of the knowledge, also of the thing. … There will be no essence belonging to anything that is

42. *Physics* 2.1, 193b7, Sachs, 51.

43. *Physics* 2.2, 194a14–15, Sachs, 52.

44. *Politics* 1.2, 1253a20–22, trans. Carnes Lord (Chicago, IL: University of Chicago Press, 1984), 37, *Meta.* 7.11, 1036b31–2, Sachs, 140; see also *Meta*. 7.10, 1035b23–5.

45. *Meta*. 5.26, 1023b33–5, Sachs, 103–4, *Meta*. 7.16, 1040b14–17, Sachs, 150, also *De Anima* 2.2, 414a20–28.

46. The exception is parts (intervals) of a line potentially present in the line, *Phys.* 8.8, 263a28.

> not a species (εἶδος) of a genus. … [B]y form (εἶδος) I mean the essence of each thing.[47]

In particular, our ability to know what is going inside a natural substance ("to see all the wheels functioning") is limited.

Aristotle's species-specific, form-limited holism stands at the opposite extreme to Newtonian species-neutral reductionism. Is there a reasonable mean? The reality of laws of nature and the particular successes of reductionist explanation make clear that nature is less Aristotelian than Aristotle thought. But classical physics suffered its own great embarrassment on the problem of the stability of matter, the problem of how to account for the specific properties of atoms and molecules (i.e., of the chemical species).

6 Stability of Matter: Radical Failure of Classical Physics

Rutherford's analysis of scattering experiments in 1911 led to the nuclear or "planetary" model of the atom: a dense, positively charged nucleus surrounded by smaller electrons in a much larger environing space. It seems, at first (conceptual) glance, like a tiny solar system (hence the name "planetary") and thus like a vindication of Newton's universal forces and particles model.[48] Most important, it seems like a perfect example of classical physics as described by de Broglie: a system exhibiting continuity, imageability, determinism. But, as we know today, it isn't: The classical conception of the nuclear atom posed the problem of the stability of matter, leading to one of the great scientific revolutions of the twentieth century, quantum physics. The problem of the stability of matter has two parts, internal and external, as follows.

6.1 The Problem of Internal Atomic Stability

Consider an isolated atom. Negatively charged electrons must be strongly attracted (by the Coulomb force) to the positively charged nucleus (containing protons and neutrons). What maintains the electrons in their orbits around the nucleus against the electrical force that pulls them in to the nucleus? For, unlike an orbiting planet in Newtonian gravitational theory, an orbiting charged particle in classical electromagnetic theory must emit electromagnetic radiation, which depletes its kinetic energy; continuous acceleration along its classical trajectory entails continuous

47. *Meta.* 5.17, 1022a5–10, Sachs, 99, *Meta*. 7.4, 1030a12–13, Sachs, 122, *Meta*. 7.7, 1032b2, Sachs, 128, translation modified slightly.

48. E. Rutherford, "The Scattering of α and β Particles by Matter and the Structure of the Atom," *Philosophical Magazine* 21 (April 1911), 669–88. The characteristic radius of the solar system is 10^{12} meters, of a hydrogen atom 10^{-10} meters. The assumption that the same principles should hold on both scales is extraordinary.

energy loss by the moving particle, which accordingly spirals into the nucleus in a flash, as described in standard textbooks on modern physics.[49] In other words, executing the trajectory calculation of Part 2, above, using the laws of electromagnetism for the electron's motion around the nucleus beginning from initial position and velocity, $\mathbf{r}_0$, $\mathbf{v}_0$, as we did using Newton's law of gravitation for a planet's motion around the sun, leads, not to a spectacular success, but a radical failure. Neither can the electrons maintain fixed positions about the nucleus (another model briefly considered) because there is no stable equilibrium configuration of static, electrically charged particles, a result derivable from Laplace's equation, $\nabla^2\phi = 0$, for the electric potential in free space.[50]

An electron in an atom does not behave according to the properties that define it classically—mass, charge, and (the classical electron) radius—and that would determine classically its local motion in an electric field. Rather, the phenomena of atomic stability require that the electron be altered, modified, in some way limited by its membership in the whole atom. Is this not a distant echo of Aristotle's obscure doctrine of material parts potential in the informed whole?[51]

Aristotle's obscurity will be removed by the mathematical theory of quantum states, their superpositions and transformations, but that theory removes as well the central concepts of classical physics: particle trajectory and field magnitude (spatio-temporally continuous and deterministic); they are not adequate to nature. It remains to discuss the stability of atoms against external disturbances.

6.2 The Problem of External Atomic Stability

Consider the many atoms composing a liquid or a solid, atoms so closely packed that, unlike a gas, the materials they compose resist compression. What enables each atom to maintain its shape and integrity, thus its specific characteristics, against the strong external disturbances (crunching against the other atoms or, if they are on the surface of the liquid or solid, being buffeted by light) to which it must be continually exposed?

The failure of classical physics on the external stability problem is succinctly described by Niels Bohr as paraphrased by Werner Heisenberg in the following

49. See, for example, R. M. Eisberg, *Fundamentals of Modern Physics* (New York: John Wiley & Sons, 1961), 108–9, also 366–9, on the great importance of the Pauli exclusion principle, namely, "[i]n a multi-electron atom there can never be more than one electron in a given quantum state" (366); without this, "[a]toms, and therefore the entire universe, would be *radically different*" (368).

50. See: Thomson and Tait, *Treatise*, 372–3; L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields* (Reading, MA: Addison-Wesley, 1962), 100; Feynman, vol. II, §5–2, especially p. 5–4, "Stability of Atoms."

51. The notion of parts existing potentially in the whole is complemented by the doctrine of parts being virtually present in the whole: the electron retains its mass, m_e, and charge, q_e, in an atom (this is the unmodified presence), but the function of the electron is not given by the classical equations for a particle of mass, m_e, and charge, q_e, in the field of the nucleus; what the particle is when it is inside the atom is thus modified relative to its nature as classically conceived.

excerpt, which situates the difficulty precisely in the notion of deterministic particle trajectory:

> My starting point was not at all the idea that an atom is a small-scale planetary system and as such governed by the laws [like those] of astronomy. I never took things as literally as that. My starting point was rather the stability of matter, a pure miracle when considered from the standpoint of classical physics.
>
> By 'stability' I mean that the same substances always have the same properties, that the same crystals recur, the same chemical compounds, etc. In other words, even after a host of changes due to external influences, an iron atom will always remain an iron atom, with exactly the same properties as before. This cannot be explained by the principles of classical mechanics, certainly not if the atom resembles a planetary system. Nature clearly has a tendency to produce certain forms … and to recreate these forms even when they are disturbed or destroyed. You may even think of biology: the stability of living organisms, the propagation of the most complicated forms which, after all, can exist only in their entirety. But in biology we are dealing with highly complex structures, subject to characteristic, temporary transformations of a kind that need not detain us here. Let us rather stick to the simpler forms we study in physics and chemistry. The existence of uniform substances, of solid bodies, depends on the stability of atoms; that is precisely why an electron tube filled with a certain gas will always emit light of the same color, a spectrum with exactly the same lines. All this, far from being self-evident, is quite inexplicable in terms of the basic principles of Newtonian physics, according to which all effects have precisely determined causes, *and according to which the present state of a phenomenon or process is fully determined by the one that immediately preceded it*. This fact used to disturb me a great deal when I first began to look into atomic physics.[52]

Classical theory makes unintelligible (or a matter of extraordinary improbability, "a pure miracle") health and healing ("the stability of living organisms"), and, more to the point, the *ground state* characteristic of a given species of atom, to which the atom returns by emission of specific frequencies of light after it is disturbed by an external influence.

We can see exactly what Bohr is getting at by means of the trajectory calculation (§2 above): Imagine that a typical classical system, the solar system, suffers a strong external disturbance. Say a large comet or asteroid passes through the solar system, not colliding with any planets, but pulling them off of their previous orbits through its own gravitational force. Is there anything in the fundamental principles of Newtonian physics—the principles that we used to calculate a trajectory from given initial conditions—that would cause the planets to recover their previous orbits? The answer is, no, for the effect of the comet or asteroid is

52. Werner Heisenberg, *Physics and Beyond*, trans. A. J. Pomerans (New York: Harper Torchbooks, 1972), 39; emphasis added.

simply to "reset" the initial conditions, the positions and velocities of the planets, which thereafter *fully* determine the future trajectories under the laws of motion and force. There is no room in this classical kind of reasoning for the solar system somehow to remember, as it were, its past configuration and get back to it.

The radical species-neutrality and reductionism of the classical world conception make nature completely indifferent to itself, thus without privileged states that are specific to the kind and self-reconstitutive—like the ground states of the atoms of the chemical elements. This is the world-historic failure of classical physical theory: it cannot account from its own first principles for the evident specificity of the material world.[53]

I note that, as elemental, a given kind of atom enters into and is common to many species of more composite bodies (e.g., carbon is an essential element in all living bodies). As such, the chemical elements are themselves species-neutral principles of nature and natural science. More fundamentally, however, the quantum physics that accounts for the specific stability (ground state, allowed transitions, chemical bonds and reactivity) of atoms is species-specific and thus holistic, unlike classical physics.[54]

In sum: In view of the two-fold problem of atomic stability, it is not surprising that the new type of theory required to account for the phenomena does not possess the three fundamental characteristics of all classical physics: (1) continuity of space, time, and motion, (2) spatio-temporal imageability of elementary processes, and (3) deterministic causality. The next step in this story would be quantum physics and its non-classical characteristics.[55] This vast topic is very well covered in many

53. To be sure, for about two centuries, classical physics solved all sorts of engineering problems in which the stable properties of liquids and solids were taken for granted and incorporated in the equations as boundary conditions or empirically determined constants. For example, water is incompressible and has a given viscosity, while cement is solid, unlike butter, and will contain the water in a swimming pool, whose surface will be horizontal in equilibrium in the earth's gravitational field. If disturbed, the water will propagate surface waves and eventually return to its stable equilibrium state with a flat surface. This does not, however, explain the respective characteristics of water and concrete in terms of their atomic and molecular constituents, or nuclei and electrons. For this, quantum physics is required.

54. A good question is posed: as we ascend from the more elementary and potential to the more composite and actual levels of nature (from nuclei and electrons to atoms, to molecules, to gases, liquids, solids, cells, tissues, organs, organisms) where does the holism "top out," i.e., at what level do we have a whole complete in itself and independent of some larger whole; at what level do we have a *substance*? Aristotle's answer is commonsensical: "a human being or a plant or something of that sort … we most of all call substances." *Meta*. 7.7, 1032a20. But there is more to the story as, e.g., Spinoza and Hegel make clear, not to mention Aristotle's own account of the substance that is most fully actual (*Phys*. 8.10, *Meta*. 12.7).

55. The continuous but non-spatio-temporal time-evolution of the wave function punctuated by its discontinuous "collapse" in the act of measurement, superposition and interference of probability amplitudes, Heisenberg indeterminacy and Bohr complementarity, non-locality or entanglement (note 8, above).

works,[56] and lies outside the natural-philosophic and phenomenological intention of the present essay. Accordingly, I return to the mathematization of nature, which extends to both classical and quantum physics, and I take up physico-mathematical secularism, which succeeds for classical physics but fails in quantum physics.

7 Physico-Mathematical Secularism: Working around the Question of the Difference between Mathematical Objects and Physical Objects

Are mathematical objects different in some fundamental way from physical objects? Plato says: yes—as intelligible, mathematical objects must exist independently of all sensible (material and changeable) things. Aristotle says: yes, but not in the way that Plato thinks—mathematical objects do not exist independently of sensible things but they can be understood independently of them through abstraction.[57] Descartes says: no, the object of physics, matter in motion, *is* the object of geometry, figurate extension.[58] My point is simply that there are well-known, major disagreements in the history of philosophy about the being of mathematical objects and their relation to physical objects. In the face of these long-standing disagreements, Newton represents a new position: Let us set aside these philosophical disputes, and *assume that any difference between mathematical objects and physical objects makes no difference for the conduct of our mathematical physics.* Henceforth, one can have one's private beliefs about the modes of being of mathematical and physical objects, such as central forces, but no scientific attention will be paid to the question. It will suffice to focus on the mathematical principles of natural philosophy; other principles (and causes) need not be discussed. Newton does not explicitly say the italicized words, above, but his posture is fairly clear from the content (not to mention the title) of the *Principia*, especially the Preface with its theme of "accuracy."[59]

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56. For extensive bibliography (covering all interpretations, not just Copenhagen) see Bryce S. Dewitt and R. Neill Graham, "Resource Letter IQM-1 on the Interpretation of Quantum Mechanics," *American Journal of Physics* 39 (July 1971), 724–36, and John A. Wheeler and Woyciech H. Zurek, eds., *Quantum Theory and Measurement* (Princeton, NJ: Princeton University Press, 1993). For a good account written for laymen, see David Lindley, *Where Does the Weirdness Go?* (New York: Basic Books, 1996), especially 129–68. For a useful article-length overview, see Max Tegmark and John A. Wheeler, "100 Years of Quantum Mysteries," *Scientific American*, February 2001, 68–75.

57. For example, Plato, *Republic*, 510c–e, 525d–e, 529b, Aristotle, *Physics*, 193b23–194a2, *De Anima*, 431b13–18.

58. See note 22, above.

59. In Def. VIII, concerning central forces, Newton says, "I here design only to give a mathematical notion of those forces, without considering their physical causes and seats." *Principia*, 5. The Preface of the *Principia* is more subtle (and remarkable): The first third explains that both mechanics and geometry should be understood in terms of (and subsumed under) *accuracy*. Accuracy means the fit or match between (1) a perfect figure ("perfectly accurate") and one drawn less perfectly, as well as (2) the closeness of a calculated to a measured number of units ("accurately proposes and demon-

I explicate the meaning of physico-mathematical secularism in terms of (1) Cartesian coordinates in the classical framework of space and time, (2) the Heisenberg uncertainty principle in quantum physics, and (3) act and potency in the physics of Aristotle.

Physico-mathematical secularism is embedded in the use of Cartesian coordinates (x, y, z, t) to represent *physical* space and time, and physical properties of bodies, particles, and fields. Consider the particle trajectory that we calculated and represented on paper in §2: At each instant of time, t, the particle is conceived to possess a real-numerically precise value of position, x, y, z, and a real-numerically precise value of momentum, mv_x, mv_y, mv_z, relative to the center of force. In general, it is assumed that such variable magnitudes can faithfully represent anything measurable in the physical world. It is thus presupposed that the properties of physical objects are conformable to the real-number line, or that the real-number line is perfectly adequate to the properties of physical objects.[60] Our measurements (using instruments of increasing precision) will match (with increasing accuracy) our calculations. We can then predict, and, to the extent possible, control the physical quantities that we conceive as objectively existing in space and time (e.g., particle position and momentum). Or are we confusing mathematical objects with physical objects? No matter (pun intended); it is not a problem: Unwitting reification of mathematical objects can do no harm (can lead to no fundamental error) in physics—this follows from the original (seventeenth-century) assumption of physico-mathematical secularism.[61]

strates the art of measuring"), i.e., numerical precision in terms of decimal digits, as in "accurate to the fifth decimal place" (see Klein, "On Precision," note 13, above). Thus when Newton (remarkably) says, "the errors are not in the art, but in the artificers," I take him to mean that there is no mismatch between the mathematical and the physical *that is rooted in the nature of the physical itself.*

60. The real number system was brought to explicit definition in the nineteenth century through the work of Dedekind and Cantor, and so Descartes and Newton did not use the terms "real number" or "arithmetical continuum." The absence in their work of any concern for the long-standing premodern doctrine of the essential heterogeneity of discrete number and continuous magnitude makes clear that their conception of number and magnitude was implicitly real-numerical, e.g., the variable, x, is both a number and a line segment, thus the expression, $1 + x + x^2 + x^3 + x^4 + \ldots$ ($x<1$), as in Newton's analyses of infinite series, makes sense not only arithmetically but also geometrically as it could not in classical geometry.

61. The significance of non-linear dynamics or "chaos theory" is that, while leaving the classical concept of deterministic trajectory intact, it corrects the assumption of the unconditional conformability of measurable properties of physical objects to the mathematical real number line. *As measurable*, physical properties can be numerically specified only approximately, e.g., to within Δx, finite no matter how small. Therefore, if a non-linear physical system is in the regime of sensitive dependence on initial conditions, then, due to the rapid divergence of initially adjacent (within Δx) trajectories, no actual measurement, e.g., of initial position, $x(t_0) \pm \Delta x$, can determine a unique trajectory. "Newtonian dynamics has, over the centuries, twice foundered on the assumption that something was infinite when in fact it was not: the speed of light, c, and the reciprocal of Planck's constant, 1/h. Reformulations omitting these infinities led first to special relativity and then to quantum mechanics. Complexity theory now reveals a third tacitly assumed infinity in classical dynamics, namely the assumption of infinite computational and observational precision." Joseph Ford, "How Random is a

This assumption is dubious, the more so in view of the highly constructed character of the Cartesian, numerical-variable magnitudes that we use in our physics. To see what I mean by "highly constructed character," look at the two magnitudes (lines) shown in boldface in Figure 3. Are the two equal or unequal? The answer is: yes. As Euclidean geometrical magnitudes, they are obviously unequal, since I can lay the left one adjacent to the right one, cut off a part of the right one equal to the left one, and exhibit the remainder. (*Elements*, common notions 4 and 5). But as Cartesian variable magnitudes, taken in our minds as possessing (unlike Euclidean magnitudes) *numerical* value, point by point, based on our arbitrary choice of a unit length, they are obviously equal, since they both have the same numerical value, c. We, unlike pre-modern mathematicians, have two different concepts of magnitude in our mental toolbox. The difference between them is considerable.

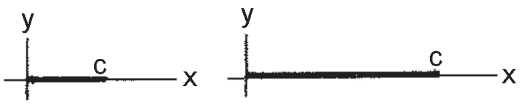

Figure 3 Two kinds of geometrical magnitude

The continuous magnitudes of Euclidean geometry are "put before us" in a direct way:[62] A line, for example, is delimited by its two endpoints (each designated by a letter) within a given figure, and it is the definition or *logos* of the figure from which the intelligibility or meaning, thus the *being* of the line—how it is related (by equality, proportionality, parallelism, perpendicularity, etc.) to the other parts of the figure—*begins*. The line has no numerical character; it is not a *length*, i.e., a number of units of measurement. Nor are two points separated by a *distance* (the length of the line they delimit). Accordingly, Euclidean demonstrations do not involve the operations (+, –, ×, ÷, √) to which numbers are subject.[63] Finally, we must continually look with our eyes at the drawn figure in front of us and examine it, but we know that our thinking is not about that figure but about an ideal, intelligible one, of which the visible one is an imperfect image.

The numerical-variable magnitudes or *coordinates* of Cartesian analytic geometry are quite different. Here, everything begins (consider plane problems) from

Coin Toss?," *Physics Today*, April 1983, 40–47, here 46. Another point, more centrally relevant to my purpose: If physico-mathematical secularism arose in the seventeenth century, what about the preceding Aristotelian–Scholastic physics and mathematics, which spoke of mixed sciences; what is the relation between the mixed-science tradition and physico-mathematical secularism? This important question is discussed in the conclusion.

62. I owe to Andrew Romiti the very helpful phrase "put before us."

63. More precisely: Euclidean magnitudes of the same kind can be added unconditionally. They can be subtracted on condition that the one removed does not exceed that from which it is taken. "Multiplication" (called "application") is limited to construction of a rectangle from two lines or a solid from a line and a plane figure. Division of heterogeneous magnitudes (a rectangle by a line) is impossible. Ratio can exist only between two magnitudes of the same kind; accordingly, a proportion involving heterogeneous magnitudes cannot be alternated.

the x and y *axes*. A point on an axis (say, the x-axis) is conceived as having, not a particular numerical value (e.g., 53.74), but numerical value in general, x, which stands for the distance from the origin that it would have if a unit of length were chosen on the axis. The origin does have a particular value: 0. Since a unit length can always be chosen (place a mark on the axis to the right of 0) whereby any line acquires a numerical value in terms of that unit (ratios of incommensurable lines having been conceived as irrational numbers), we need not choose a unit prior to performing calculational operations, +, –, ×, ÷, √, on the letter signs, x, y, a, b, etc. A unit can be specified later, if needed. We are working with generic numbers (increasing to the right on the x-axis or upward on the y-axis from the origin). The x–y plane is then brought into being by our mental conception of its points as ordered pairs; that is, as coordinates (x, y), where x means the distance the point would have from the y-axis measured parallel to the x-axis, and y means the distance it would have from the x-axis measured parallel to the y-axis upon choice of a unit. Most relevant and useful: the coordinates, x and y, are conceived *to vary* (lengthen and shorten) relative to the origin at (0, 0). We can stipulate and express how, say, y varies with x in a *functional relation*, e.g., $y(x) = ax + b$. This *equation* is usefully expressed by the curve (here a line) whose points possess the coordinates (x, y(x)) thereby defined. Curves and their equations can be studied, and problems can be solved by operating on the letter-signs and finding the algebraic expressions for the functional relation between coordinates of interest, such as the position and momentum of a particle in relation to time, x(t), p(t). The Cartesian letter-signs (or symbols, x, y, t, a, b, and so on) for variables and constants do not image ideal objects as do the figures of Euclidean geometry. Nor do they stand for the definite numbers of pure units (monads) or their fractional parts as in the Diophantine arithmetic.

My point in all of this is that the mind's constructive activity contributes much to the structure of Cartesian geometry—to the intelligibility or meaning of its thus conceptually complex objects. (Note the appearance of the words "conceived," "conception" in the preceding paragraph; they have the sense defined in Descartes' *Rules for the Direction of the Mind*.[64]) The Cartesian plane, and the coordinate axes that bring it into being are, therefore, not "put before us" in the direct manner of Euclidean magnitudes.[65] Now can we *identify* the spatio-temporal objects of physics with these conceptually constructed objects of mathematics? Is the conformability of the former (the physical) to the latter (Cartesian magnitudes), or the adequacy of the latter to the former unconditional—as was assumed by the physico-mathematical secularism of classical physics? Is there nothing in the

64. Rule 12, AT X, 416, CSM I, 42. The concept of general magnitude—the progenitor of real-numerical variable magnitude or coordinate axis—is then elaborated in Rules 13–16.

65. Using the Scholastic terminology of first and second intentions, Klein characterizes Cartesian magnitudes as second intentions taken by the intellect as first intentions by means of the visible letter sign, which thereby becomes an algebraic *symbol*, and analytic geometry an algebra of line segments. See *GMTOA*, 208, and *Lectures and Essays*, 17–21.

nature of the physical that might impede its perfect match to the mathematical? My repeated reference, above, to the position and velocity or momentum, thus the trajectory of a particle should make the answer to this question quite clear. The Heisenberg uncertainty principle is emblematic of the failure of physico-mathematical secularism.

In 1927 Werner Heisenberg set out what has since been known as the uncertainty principle of quantum physics: "The more accurately the position [of a particle] is determined, the less accurately the momentum is known and conversely."[66] More specifically: the product of the uncertainty in the measurement of particle position, Δx, at time t, and the uncertainty in the measurement of its momentum, Δp, at time t, cannot be reduced below (approximately) the numerical value of $\hbar$, Planck's constant divided by 2π:

$$\Delta x \Delta p \geq \hbar$$

The reciprocal relation of the uncertainties means that, as position, x(t), is measured with increasing accuracy—approaching the classical ideal, $\Delta x = 0$—all values of the momentum, p(t), become equally possible, and conversely, as $\Delta p \rightarrow 0$, $\Delta x \rightarrow \infty$. The Δx and Δp are not observational errors of classical type resulting from the imprecision of present-day instruments, to be progressively sharpened in the future. Most important, the Δx and Δp are not merely expressions of an unpredictable disturbance of the tiny particle's (e.g., the electron's) local motion due to the unavoidably much larger (and energetic) observational apparatus. This is the "uncontrollable disturbance" interpretation of the uncertainty principle. It leaves the central concept of classical mechanics—particle trajectory (i.e., simultaneously well-defined values of position and momentum)—intact, maintaining only that the trajectory is made unpredictable in the future by our unavoidably intrusive efforts to observe the particle right now (it gets uncontrollably deflected by the light used to detect it), so that the trajectory is uncertain only to us, not indeterminate in itself. Rather, the uncertainty principle is ontological, such that, "[t]he term 'uncertainty principle' is, therefore, somewhat of a misnomer. A better term would be 'the principle of limited determinism in the structure of matter'," or the Heisenberg indeterminacy principle.[67]

Heisenberg's principle means, among other things, that the being and knowability of spatio-temporal properties, like particle position, x, at time, t, are

66. "Je genauer der Ort bestimmt ist, desto ungenauer ist der Impuls bekannt und umgekehrt." Heisenberg, "Über den anschaulichen Inhalt der quantentheoretischen Kinematic und Mechanik," *Zeitschrift für Physik* 43 (1927), 127–98, here 175.

67. David Bohm, *Quantum Theory* (New York: Dover, 1979), 101. For the quantum mechanical explanation of a track in a cloud chamber, see 137–40. Heisenberg indeterminacy and Bohr complementarity are the essential features of the Copenhagen Interpretation of quantum mechanics. Alternative interpretations that accord with the classical conception of mind and world are, after Bell's theorem and the Aspect experiments, necessarily non-local and, in the case of Bohm theory, radically holistic; see D. Bohm and B. J. Hiley, *The Undivided Universe* (London: Routledge, 1993).

intertwined with the being and knowability of dynamical properties, like momentum, p, and energy, E. Here, again, is de Broglie:

> What is now [e]specially important for us to understand is the profound meaning of this rather mysterious idea of the quantum of action [Planck's constant, h = 6.62×10^{-27} erg-sec]. Up till [the early twentieth century] the space and time of classical physics, or its successor—the space-time of the relativity physics—had appeared to us as a framework given *a priori* and [being] quite independent of what one could put into it, [being] quite independent particularly of the movements and evolution of the bodies which were localized in it. …
>
> The real significance of the quantum of action has been disclosed to us notably by the discovery of Heisenberg's uncertainties. … It seems certain today that the existence of the quantum of action expresses a formerly totally unsuspected union between the framework of space and time and the dynamical phenomena which take place in it. The picture of space and time [in classical physics] is essentially static; a body, a physical entity, which has an exact location in space and in time is, by this very fact, deprived of all evolutionary property; [but] on the contrary, a body which is developing, which is endowed with dynamic properties, cannot really be attached to any point of space and time. These are philosophical remarks which go back to Zeno [and so to Aristotle, *Phys.* 8.8]. … Heisenberg's uncertainty relations appear akin to these remarks; they teach us, in effect, that it is impossible to attribute simultaneously to a body a well-defined motion and a well-determined place in space and time.[68]

There is no way in the classical conception of mind and world, space and time, mathematics and physics, measurement and calculation, in which our knowledge of one physical quantity, say, position or time, could affect or interfere with, or limit our knowledge of another such quantity, say, momentum or energy. They are all just Cartesian magnitudes, real numbers of appropriate units. They can be thought conjointly and put together on the paper (the position and velocity vectors in the trajectory calculation); or separated in our thought and on paper. As Cartesian mathematical objects, each is unmodified by, and indifferent to its membership in any whole. There is no "holism" of motion, mobile, place and time. But, as de Broglie reminds us, pointing back to Aristotle, a moving body—as opposed to a mathematical point "moving" in our imagination—is not *actually* in a place or at a fixed position; if it were, it would not be in motion. Matt Crawford puts it nicely:

> The identification of natural beings, having first intentional magnitudes (as Klein says), with Cartesian real-numerical magnitudes (by which identification they are also reduced to a collection of decoupled, quantity-holding attributes, abstracting from other … characteristics that depend on the coupled whole), is no longer tenable given that, as we now know, the framework of space and

68. De Broglie, *Microphysics*, 120–22.

> time is only putatively determinate in the way real numbers are, and this putative determinateness is an artifact of its abstract conception.[69]

Aristotle, of course, did not quantify the indeterminacy in the position of a body moving with a given speed; he did not discover Planck's constant. But Aristotle does prepare us for the ideas of indeterminacy and potentiality—of some things having less being than others—and thus of limits to the intelligibility of the potentially being. Heisenberg used Aristotle as an aid in attempting to explain the obscure reality of the wave function: it is "a quantitative formulation of the concept of δύναμις, possibility, or in the later Latin version, *potentia*, in Aristotle's philosophy."[70] The actualization of this potentiality is the act of measurement whereby the wave function is reduced, or "collapses" to one of the *eigenstates* of which it is a superposition (a type of sum) and to which corresponds a possible (potential) result of the measurement (e.g., the particle has spin up or spin down). Heisenberg's analogy to Aristotle is right in this sense: The process of wave-function collapse cannot be expressed mathematically, it cannot be modeled (imaged) as the spatio-temporal variation of some Cartesian magnitudes as in a classical equation of motion. Wave function collapse from a set of many possible outcomes to one actual outcome escapes the grasp of mathematics. With a notable exception, discussed below, the reduction of *dunamis* to *energeia* in Aristotle's physics likewise escapes the grasp of mathematics. Unlike Aristotle's act and potency, however, the possible outcomes of a quantum measurement occur with numerical probabilities that can be calculated from the wave function (e.g., 0.75 for spin up, 0.25 for spin down). More generally, of the various senses of the potentially being and its actualization in Aristotle, none fits quantum processes exactly.[71]

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69. Matt Crawford, private communication, July 30, 2006.

70. The full quotation is that the probability amplitude, $\psi(x, t)$, is "a quantitative formulation of the concept of δύναμις, possibility, or in the later Latin version, *potentia*, in Aristotle's philosophy. The concept that events are not determined in a peremptory manner, but that the possibility or 'tendency' for an event to take place has a kind of reality—a certain intermediate layer of reality, halfway between the massive reality of matter and the intellectual reality of the idea or the image—this concept plays a decisive role in Aristotle's philosophy. In modern quantum theory this concept takes on a new form; it is formulated quantitatively as probability and subjected to mathematically expressible laws of nature [e.g., the Schrödinger equation]." Werner Heisenberg, *On Modern Physics* (New York: Clarkson Potter, 1961), 9–10.

71. I find five senses (some overlapping) of potentiality with respect to material structure and change in Aristotle: (1) motion (κίνησις) in the categories of quantity, quality, and place, and (2) change (μεταβολή) in the category of substance (*Phys.* 3.1, 201a11, 210a29, 201b5, 3.2, 202a7, *Meta.* 5.12, 1019a21–3, 7.9, 1034a35–b1, 9.7, 1049a14–17); actualization of (3) first and (4) second potency (*Phys.* 8.4, 255a32–b14, *De An.* 2.1, 412a22–9); (5) body parts being potentially in a whole living substance (*Meta.* 5.26, 1023b33–5, 7.16, 1040b5–17, *De An.* 2.2, 414a20–28). With respect to mathematical objects, points and intervals can be potentially in a line (*Phys.* 8.8, 262a22–4, 263a28). For a detailed and clear explanation of the sense of potentiality in quantum physics, see Bohm, *Quantum Theory*, 132–3, 138–40, 166–7. This is not to be confused with the quantum potential of Bohm's later hidden-variables (specifically pilot-wave) theory; see "A Suggested Interpretation of the Quantum Theory in Terms of 'Hidden' Variables," *Physical Review* 85, no. 2 (January 15, 1952): 166–93.

Let us state the essential conclusion of this section: Because act and potency, thus actualization of potentiality and material parts potential in the whole, do not belong to mathematical objects—whether ancient or modern—modern physico-mathematical secularism entails their banishment from physics. This decision was adequate for classical physics, but not for quantum physics.

Conclusion: Seeing Heterogeneity, Making Homogeneity

The preceding paragraph poses a final question on which to collect our thoughts and finish this essay: Pre-modern (ancient and medieval) physics had a significant mathematical component, namely, the four mixed or intermediate sciences, optics, harmonics, mechanics, and astronomy, which employed pre-modern mathematics, namely, arithmetic, geometry (also trigonometry), and proportion theory.[72] For example, Aquinas speaks of "reckoning the courses of the stars" (*cursus siderum computare*) in mathematical astronomy, for "mobile and incorruptible beings, owing to their uniformity and regularity, can be determined in their movements by mathematical principles."[73] But there is nothing in pre-modern science like physico-mathematical secularism, no suspension of judgment concerning the difference between mathematical objects and physical objects, thus no ruling (methodological) assumption that the difference between them could make no difference for the science of nature. What accounts for this? The answer to this question becomes clear to the extent that we can remove ourselves from the conceptual standpoint of modern mathematics and physics and recover that naively direct (and thus error-prone) way of receiving the world characteristic of Aristotle's philosophy. There we see (literally and figuratively) *heterogeneity*—differences in kind—in both mathematical beings and physical beings.

As long as we apprehend several different kinds of mathematical beings (continuous magnitudes, discrete numbers, and, where homogeneity restrictions permit, ratios and proportions of them) none of which is subject to κίνησις, along with several other different kinds of physical beings (celestial, terrestrial, and among the latter, non-living and living, and among the latter, plants and animals, and among the latter, non-human and human) each of which is substance and subject to κίνησις in different analogous degrees—as long as all of this is the case, it is clearly impossible to think that the difference between mathematicals and physicals might not matter for the science of nature.

The most striking visible difference or heterogeneity in the physical world is that between celestial and terrestrial. Aquinas just mentioned the most perfect

72. Aristotle, *Post. An.*1.13, 78b35–79a16, *Phys*. 2.2, 193b32–194a12.

73. Aquinas, *In de trinitate*, q. 5, a. 1, ad 3, and a. 3, ad 8, trans. Armand Maurer, *The Division and Methods of the Sciences* (Toronto, Ontario: Pontifical Institute of Medieval Studies, 1986), 18, 46. In "reckoning the courses of the stars," geometrical figures (Ptolemaic circles) would image their paths while numbers and fractions of angular units (all discrete) would approximate the positions at a given time of the stars continuously moving on those paths.

kind of physical beings, the celestial bodies, which are (by the erroneous science of that time) incorruptible, and whose local motions are, accordingly, so regular that arithmetic and geometry can be applied to them. But, therefore, by virtue of that very perfection, among the celestial bodies, generation does not occur, nor corruption, but only local motion, in which the mobile "departs least from its substance" (*Phys.* 8.7, 261a22). In striking contrast, among the terrestrial living things, there is a craft-like succession of stages in their generation from seed (*Phys.* 2.8, 199a8–19); final cause and chance are evident there, but there is (to this day) no mathematical description of their natural local motions. In sum, celestial and terrestrial beings are vividly distinct in their patterns of motion: in the one kind, we have mathematical description but final cause is not evident; in the other, final cause is evident but we have no mathematical description.

Among mathematical beings, the essential heterogeneity is that of discrete number and continuous magnitude. It is very difficult for us today to understand sympathetically the reasons for this long-standing, pre-modern distinction. We are, as it were, born to the real number line (i.e., to the Cartesian numerical-variable magnitudes), and this mental formation begins with elementary-school arithmetic. Jacob Klein's work on the origins of algebra is about this, and thus about the removal of the distinction within human cognition between discrete and continuous mathematical beings in favor of algebraic *symbols* lying in "a new [homogeneous] conceptual dimension"—and all that this implies.[74]

I wish here simply to bring out the following basic point: It is only after homogenizing the heterogeneities of both physical and mathematical objects, and thus conceiving the possibility of an adequation, fit, or match of Cartesian numerical magnitudes to the properties of physical beings, that physico-mathematical secularism becomes possible. These two homogenizations have this in common: they have more the character of making (construction, creation) than of seeing (intuition, intellection). But it seems there is also a difference: Whereas the homogenizing transition in mathematics is for us today inconspicuous (accessible only by the desedimentation accomplished by Klein), the one in physics seems conspicuously willful: Granted, it was surely right to remove the mistaken heterogeneity of celestial and terrestrial, but was it right to remove *all* heterogeneity from nature (living and non-living, *human and non-human*), thereby committing science to conceive nature as all one stuff (e.g., particles and fields, or DNA and protein)—"material to work on," in Locke's memorable phrase?[75] As long noted

74. "[This] modification … of ancient mathematics is *exemplary* for the total design of human knowledge in later times." *GMTOA*, 121. "Therewith the most important tool of mathematical natural science, the 'formula', first becomes possible … but, above all, a new way of 'understanding', [a different conception of the world, a different understanding of the world's being] inaccessible to ancient *episteme* is thus opened up." *GMTOA*, 175, [152]. See *GMTOA*, 184–5, 192, 213, on the awareness of fundamental problems that is thereby lost, e.g., the "old questions of 'one-and-many': One *over*, or *in*, or *out of*, many?" (Eva Brann, "Jacob Klein's Two Prescient Discoveries," *The New Yearbook for Phenomenology and Phenomenological Philosophy* XI).

75. Locke, *Second Treatise of Civil Government*, chap. 5, §35.

by thoughtful commentators, this leaves human being, especially the scientist, in an odd position.[76] Perhaps the greatest value of Aristotle lies not in his philosophy of nature (which is valuable, as I have tried to show), but in his ethics: "[P]recision ought not to be sought in the same way in all kinds of discourse … for it belongs to an educated person (πεπαιδευμένου) to seek just so much precision in each kind [of discourse] as the nature of the subject admits."[77] But Aristotle's educated person has a special disposition that enables perception and judgment of limits and boundaries, of due measure, one that is difficult to reconcile with that of modern natural science.[78]

I give Heidegger (almost) the last word, and what he says applies to both quantum and classical physics, and, I suspect, to the new biotechnology, as well:

> Modern science's way of representing pursues and entraps nature as a calculable coherence of forces. Modern physics is not experimental physics because it applies apparatus to the questioning of nature. Rather the reverse is true. Because physics, indeed already as pure theory, sets nature up to exhibit itself as a coherence of forces calculable in advance, it therefore orders its experiments precisely for the purpose of asking whether and how nature reports itself when set up in this way. … If modern [quantum] physics must resign itself ever increasingly to the fact that its realm of representation remains inscrutable and incapable of being visualized, this resignation is not dictated by any committee of researchers. It is challenged forth by the rule of Enframing [*Ge-stell*], which demands that nature be orderable as standing-reserve. Hence physics, in all its retreating from the representation turned only towards objects [i.e., the classical conception of mind and world] that has alone been standard till recently, will never be able to renounce this one thing: that nature reports itself in some way or other that is identifiable through calculation and that it remains orderable as a system of information [i.e., digit strings].[79]

This means that our physics has nothing to say about the unity of nature or creation.

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76. In addition to the writings of Kennington and Riezler, cited above, see the reflections on natural science and human self-understanding in Hans Jonas, Leon Kass, Charles De Koninck, C. S. Lewis, and Leo Strauss.

77. *Nic. Ethics* 1.3, 1094b13, 24–5, trans. Joe Sachs, *Aristotle's* Nicomachean Ethics (Newburyport, MA: Focus Publishing, 2002), 2. Rule 1 of Descartes' *Rules for the Direction of the Mind* is precisely targeted against this (AT X, 359–60, CSM I, 9).

78. "It is only its exactitude itself, the perfect matching of mathematically obtained results with the observable data, that science considers praiseworthy." Klein, "On Precision," *Lectures and Essays*, 306.

79. Heidegger, *The Question Concerning Technology*, trans. William Lovitt (New York: Harper and Row, 1977), 21, 23.